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SOLUTION STRATEGIES FOR STATICALLY LOADED NONLINEAR STRUCTURES

by

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Report to Sponsor:
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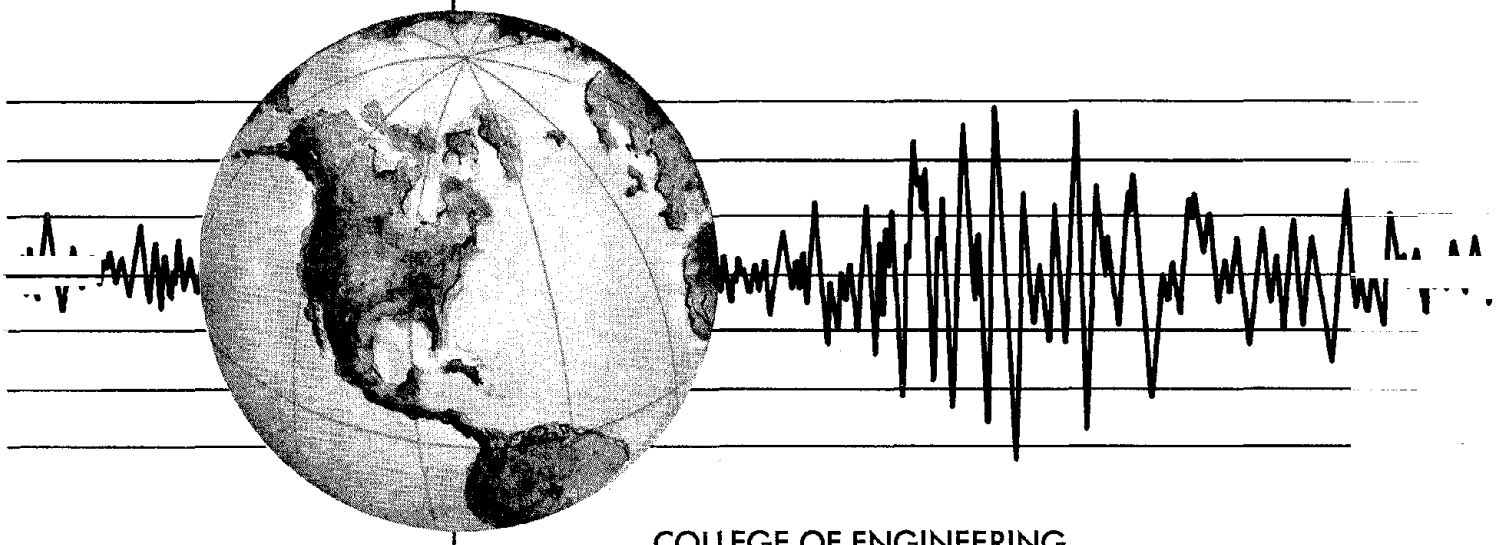

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SOLUTION STRATEGIES FOR STATICALLY LOADED NONLINEAR STRUCTURES

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ABSTRACT

Load-displacement relationships for nonlinear structures can be of a variety of types. Of the available solution strategies, none is reliable for all types of behavior, and most are reliable only for limited classes of problems. An investigation of solution strategies has been undertaken, with the aim of developing a scheme which is reliable and substantially automatic for a wide variety of statically loaded nonlinear structures.

A number of concepts and computational tools that are commonly used in nonlinear structural analysis are first identified, and a consistent terminology is suggested. The concepts of scalar force, scalar displacement and scalar stiffness are defined, and the use of rank-one stiffness and flexibility modifications is described. Next, a framework for nonlinear static solution strategies is developed, based on Newton-Raphson iteration. This framework consists of four computational tasks, namely linearization, displacement prediction, state determination, and convergence checking. These tasks are performed in two phases, namely an advancing phase and a correcting phase. Currently available solution strategies are shown to differ in the way particular tasks are performed. The strategies considered include modified Newton methods, Quasi-Newton methods, displacement-controlled iteration, techniques for variable load step selection, and event-to-event strategies. Finally, a general solution strategy is described that includes most existing schemes, plus certain new schemes as options. This strategy permits analysis by either load-controlled or displacement-controlled methods, with flexibility in the choice for the controlling parameters.

The general solution strategy has been incorporated into the ANSR-III computer program and tested on a series of four example problems. Three of the examples involve buckling of truss structures, and the fourth involves crushing of a pipe between two plattens. The examples are discussed. It is concluded that the solution strategy is reliable and efficient for a wide variety of strongly nonlinear problems.

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1. INTRODUCTION

1.1 PROBLEM AREAS IN NONLINEAR STRUCTURAL ANALYSIS

Structural engineers are increasingly faced with the need to perform nonlinear analyses to predict the true behavior of structures. Many different types of nonlinear behavior are possible, including both material and geometric nonlinearities, and numerous solution strategies have been proposed. However, no single strategy has proven to be usable, reliable, and economical for a wide variety of problem types. Some particular areas of weakness are as follows.

(1) Because different authors use different terminology, the literature tends to be confusing. There is a need for consistent terminology.

(2) Strategies are often presented as special purpose solution schemes, and it is not always obvious how they relate to other schemes. There is a need for a general framework from which to view the available strategies.

(3) Certain computational tools appear often in different schemes, although their use may not be readily apparent. There is a need to identify these tools more clearly.

(4) In general, sophisticated solution schemes require substantial skill on the part of the analyst for successful application. There is a need to simplify and automate the strategies to reduce the chance for analyst error.

1.2 OBJECTIVES OF PRESENT RESEARCH

The objectives of the present research are as follows.

(1) Clearly identify the tasks and tools commonly found in nonlinear static analysis and describe them using a consistent terminology.

(2) Construct a general framework for nonlinear static analysis, and show how the various available solution strategies fit into this framework.

(3) Devise a general solution strategy, incorporating both existing and new concepts, that will solve a variety of problems and be reliable and easy to use.

(4) Implement the strategy in the ANSR-III computer program, and test it on some strongly nonlinear examples.

1.3 REPORT LAYOUT

The objectives outlined in Section 1.2 have essentially been achieved, although there is still substantial room for improvement. Chapter 2 begins with definitions of terms and concepts used throughout the report. Newton-Raphson iteration is reviewed and a framework for nonlinear static analysis is established. Existing strategies are discussed in terms of this general framework. In Chapter 3 a general solution strategy is presented and discussed. Example problems are presented in Chapter 4, followed by conclusions in Chapter 5. Appendix A contains a user's guide, with notes, for the ANSR-III implementation of the solution strategy.

2. REVIEW AND DEFINITIONS

2.1 GENERAL

The literature dealing with nonlinear structural analysis can be difficult to interpret because no consistent terminology exists. In the following sections a suggested consistent terminology is presented, defining the terms and concepts most commonly used in static nonlinear analysis.

2.2 DEFINITIONS

2.2.1 Displacements

For analysis by the Displacement Method, the primary unknowns of the problem are a number of kinematic degrees of freedom (d.o.f.). These degrees of freedom are typically translational and rotational displacements at nodes of the structure, and will be referred to as the *structure displacements*, or simply, the *displacements*. The complete set of displacements can be arranged in a *displacement vector*, $\underline{r}$. Each element has associated with it a subset of the structure displacements, typically the displacements at the nodes to which the element connects. These are the *element displacements*, which may be arranged in an *element displacement vector*, $\underline{q}$.

The *current displacements* are the most recently calculated displacements. A *displacement increment* is a finite ($\Delta\underline{r}$) or infinitesimal ($d\underline{r}$) change in the displacements. A displacement vector can be regarded as defining a point in a multidimensional displacement space. As a structure deforms, it traces out a *displacement path* in this space.

2.2.2 Structure State

Each set of displacements corresponds to a *deformed state*, for the structure as a whole and for each element. The element deformations are related to the element displacements by shape functions which ideally (although not necessarily) ensure that geometric compatibility is satisfied. Element stresses or stress resultants *(element actions)* are related to corresponding strains or strain resultants *(element deformations)* by constitutive relationships of a variety of

possible types.

For any set of element actions there is a set of *element resisting forces,* $\underline{Q}$, which satisfies element equilibrium. These forces are external forces exerted on the element, corresponding to the element displacements. They are typically defined only in a virtual work sense. The term "force" includes both translational forces and moments. The element forces can be assembled into a vector of *structure resisting forces,* $\underline{R}_I$.

The structure displacements, element deformations, element actions and structure resisting forces constitute the *structure state.* The process of calculating the structure state for the current displacements is termed *state determination.* The calculation begins at a *reference state* and proceeds in the following steps.

(1) The displacement increment from the reference state to the current state must be given.

(2) The element deformation increments are calculated using the shape functions.

(3) The element actions in the current state are calculated, considering the reference state, the deformation increments, and the constitutive relationships.

(4) The element resisting forces are calculated by equilibrium (typically using the virtual displacements principle) and assembled to give the structure resisting forces.

The current state is an *equilibrium state* if the structure resisting forces are equal to the *external forces* on the structure, $\underline{R}_E$. The external forces consist of all applied forces and reactions acting externally on the structure, one force for each structure displacement. In a numerical solution, an exact equilibrium state will never be reached. Rather, a *converged state* will be sought in which the *equilibrium error* is acceptably small. A measure of the equilibrium error is provided by the vector of *unbalanced forces,* $\underline{R}_U$, given by:

$$\underline{R}_U = \underline{R}_E - \underline{R}_I \tag{2.2.1}$$

The criterion for convergence is commonly expressed as a tolerance on a norm of $\underline{R}_U$ (e.g. Euclidean norm, maximum absolute value).

If an iterative solution fails to arrive at a converged state, it may be necessary to restore

the previous converged state and attempt a new solution. A converged state which is saved to permit restoration is a *backup state.*

2.2.3 Stiffnesses

The structure *tangent stiffness* (or tangent stiffness matrix), $\underline{K}_T$, is defined by:

$$d\underline{R}_I = \underline{K}_T \, d\underline{r} \tag{2.2.2}$$

in which $d\underline{r}$ is an infinitesimal increment of displacement and $d\underline{R}_I$ is the corresponding increment of resisting force. The process of calculating the tangent stiffness in any state may be termed *linearization.* It is performed by calculating and assembling the tangent stiffnesses, $\underline{k}_T$, of all the elements. The tangent stiffness in the initial undeformed state, $\underline{K}_o$, is the *initial stiffness.* A stiffness, $\underline{K}_s$, which satisfies the finite relationship:

$$\Delta\underline{R} = \underline{K}_s \, \Delta\underline{r} \tag{2.2.3}$$

is a *secant stiffness.*

2.2.4 Equilibrium Equations

For a linear structural analysis, the structure displacements are typically found by solving *equilibrium equations* of the form:

$$\underline{K} \, \underline{r} = \underline{R} \tag{2.2.4}$$

in which $\underline{r}$ defines the total displacements and $\underline{R}$ is the total *load vector.* For nonlinear analysis, the displacements are generally calculated in increments, frequently (but not necessarily) by solving equations of the form:

$$\underline{K}_T \Delta\underline{r} = \Delta\underline{R} \tag{2.2.5}$$

in which $\Delta\underline{R}$ will still be termed the *load vector.*

The equilibrium equations are most often solved by direct elimination (e.g. Gauss, Crout, Cholesky), but may also be solved by iteration (e.g. Gauss-Seidel).

2.2.5 Loads

Displacement of a structure may be caused by *applied loads* of a variety of types. A static applied load is conveniently constructed as a combination of a number of separate *load patterns*, each multiplied by a *load pattern magnitude*. The set of load pattern magnitudes constitutes the *load magnitude*. A load pattern may define *nodal loads, element loads, initial strain loads,* or *imposed displacement loads.*

Nodal loads and element loads may consist of point forces, line forces, surface forces, and/or body forces. A set of forces constitutes a nodal load if its contribution to the load vector, $\Delta \underline{R}$, can be determined without considering the stiffness and/or strength properties of the elements (e.g. point forces applied directly on a node). A set of forces constitutes an element load if the element properties must be considered in setting up $\Delta \underline{R}$ (e.g. distributed load along the length of a beam element, which produces "fixed end" forces on the nodes). Nodal and element loads contribute forces directly to both the load vector, $\Delta \underline{R}$, and the external force vector, $\underline{R}_E$.

Initial strain loads (e.g., temperature change) contribute to the load vector, $\Delta \underline{R}$, but do not contribute directly to the external force vector, $\underline{R}_E$. As with element loads, the contributions to $\Delta \underline{R}$ depend on the stiffness and/or strength properties of the elements.

With imposed displacement loads, displacement increments are specified in particular directions at particular nodes. That is, in the equilibrium equation:

$$\underline{K}_T \Delta \underline{r} = \Delta \underline{R} \tag{2.2.6}$$

certain terms in $\Delta \underline{r}$ are specified, and the corresponding terms in $\Delta \underline{R}$ are initially unknown. This is taken in to account in the solution of the equilibrium equations. The nodal forces corresponding to the imposed displacements become external forces (in effect, reactions) on the structure, and hence contribute to $\underline{R}_E$.

A load pattern may be *fixed* or *configuration dependent.* The loads in a fixed pattern are independent of the displacements (e.g. forces with fixed magnitudes and directions). Configuration dependent loads vary as the structure deforms (e.g. hydrostatic pressure).

2.3 GENERAL CONCEPTS AND TECHNIQUES

2.3.1 General

A number of concepts and computational techniques are of general use in nonlinear structural analysis. Several of these are reviewed in this section.

2.3.2 Scalar Displacement and Scalar Force

The projection of a displacement vector, $\underline{r}$, on a *unit vector,* $\underline{b}$, in displacement space defines a *scalar displacement,* r_b, along the direction $\underline{b}$. That is,

$$r_b = \underline{b}^T \underline{r} \tag{2.3.1}$$

Similarly, the projection of a force vector, $\underline{R}$, on a unit vector $\underline{b}$ in force space defines a *scalar force* along the direction $\underline{b}$. That is,

$$R_b = \underline{b}^T \underline{R} \tag{2.3.2}$$

2.3.3 Scalar Stiffness

A *scalar stiffness,* K_b, is the ratio of a scalar force to a scalar displacement. That is,

$$K_b = \frac{\underline{b}_1^T \Delta \underline{R}}{\underline{b}_2^T \Delta \underline{r}} \tag{2.3.3}$$

in which $\underline{b}_1$ and $\underline{b}_2$ are unit vectors. Usually, $\underline{b}_1$ and $\underline{b}_2$ will be the same. The direction of interest will usually be the direction of either the displacement increment or the load increment. For these two cases, tangent scalar stiffnesses are calculated as follows.

In the direction of the displacement increment:

$$\underline{b} = \frac{\Delta \underline{r}}{(\Delta \underline{r}^T \Delta \underline{r})^{1/2}} \tag{2.3.4}$$

and hence,

$$K_r = \frac{\underline{b}^T \Delta \underline{R}}{\underline{b}^T \Delta \underline{r}} = \frac{\Delta \underline{r}^T \Delta \underline{R}}{\Delta \underline{r}^T \Delta \underline{r}} \tag{2.3.5}$$

which is equivalent to:

$$K_r = \underline{b}^T \underline{K}_T \underline{b} \tag{2.3.6}$$

In the direction of the load increment:

$$\underline{b} = \frac{\Delta \underline{R}}{(\Delta \underline{R}^T \Delta \underline{R})^{1/2}} \tag{2.3.7}$$

and hence,

$$K_R = \frac{\Delta \underline{R}^T \Delta \underline{R}}{\Delta \underline{R}^T \Delta \underline{r}} \tag{2.3.8}$$

which is equivalent to:

$$K_R = 1/(\underline{b}^T \underline{K}_T^{-1} \underline{b}) \tag{2.3.9}$$

2.3.4 Stiffness Ratio

A measure of the change in stiffness during an analysis can be expressed as the ratio of a scalar stiffness in the current state to a scalar stiffness in the initial state. For example, a *scalar stiffness ratio*, S_r, can be calculated as:

$$S_r = \frac{(\Delta \underline{r}^T \Delta \underline{R} / \Delta \underline{r}^T \Delta \underline{r})}{(\Delta \underline{r}_o^T \Delta \underline{R} / \Delta \underline{r}_o^T \Delta \underline{r}_o)} \tag{2.3.10}$$

in which

$$\Delta \underline{r} = \underline{K}_T^{-1} \Delta \underline{R}$$
$$\Delta \underline{r}_o = \underline{K}_o^{-1} \Delta \underline{R}$$

and $\Delta \underline{R}$ is constant. Alternatively, a scalar flexibility ratio can be calculated as:

$$F_r = \frac{(\Delta \underline{R}^T \Delta \underline{r} / \Delta \underline{R}^T \Delta \underline{R})}{(\Delta \underline{R}^T \Delta \underline{r}_o / \Delta \underline{R}^T \Delta \underline{R})} = \frac{\Delta \underline{R}^T \Delta \underline{r}}{\Delta \underline{R}^T \Delta \underline{r}_o} \tag{2.3.11}$$

Bergan [1,2] introduced the concept of a *current stiffness parameter*, S_p, to help control the non-linear solution strategy. This is the inverse of the scalar flexibility ratio defined by Eqn. (2.3.11). This parameter has the following properties.

(a) The initial value of S_p is one. Values greater than one indicate that the structure is stiffer than it was initially, and values less than one indicate that it is more flexible.

(b) Generally, for a stable structure S_p is positive, whereas for an unstable structure it is negative.

(c) Where the load magnitude reaches a local maximum, the value of S_p is zero.

(d) The rate of change of S_p is related to the nonlinearity of the response. For structures that are nearly linear S_p changes slowly, whereas for structures that are highly nonlinear S_p changes rapidly.

2.3.5 Rank One Symmetric Stiffness and Flexibility Matrices

Let $\underline{u}$ be a unit vector in displacement space, and let G be a scalar stiffness coefficient. The stiffness matrix

$$\underline{K}_u = \underline{u}\, G\, \underline{u}^T = G\, \underline{u}\, \underline{u}^T \tag{2.3.12}$$

is a rank one matrix (i.e., only one nonzero eigenvalue) which has the following properties:

(a) Any displacement in the direction of $\underline{u}$, say $c\underline{u}$, is resisted by a force in the same direction. That is,

$$\underline{K}_u\, c\, \underline{u} = G\, c\, \underline{u}\, \underline{u}^T \underline{u} = G\, c\, \underline{u} \tag{2.3.13}$$

(b) Any displacement in a direction orthogonal to $\underline{u}$, say $\underline{u}_o$, is not resisted by any force. That is,

$$\underline{K}_u\, \underline{u}_o = G\, \underline{u}\, \underline{u}^T \underline{u}_o = \underline{0} \tag{2.3.14}$$

Similarly, let $\underline{U}$ be a unit vector in force space, and let H be a scalar flexibility coefficient. The rank one flexibility matrix:

$$\underline{F}_U = \underline{U}\, H\, \underline{U}^T = H\, \underline{U}\, \underline{U}^T \tag{2.3.15}$$

has the following properties:

(a) Any force in the direction of $\underline{U}$, say $c\underline{U}$, produces a displacement in the same direction. That is,

$$\underline{F}_U\, c\, \underline{U} = H\, c\, \underline{U}\, \underline{U}^T \underline{U} = H\, c\, \underline{U} \tag{2.3.16}$$

(b) Any force in a direction orthogonal to $\underline{U}$, say $\underline{U}_o$, produces no displacement. That is,

$$\underline{F}_U \underline{U} = H\, \underline{U}\, \underline{U}^T \underline{U}_o = \underline{0} \tag{2.3.17}$$

2.3.6 Rank One Stiffness Modification

A rank one stiffness matrix can be added to a structure stiffness matrix, $\underline{K}$, to change the stiffness in a particular direction, $\underline{u}$, by an amount G. The modified stiffness matrix, $\underline{K}_m$, is given by:

$$\underline{K}_m = \underline{K} + G\,\underline{u}\,\underline{u}^T \tag{2.3.18}$$

The inverse of the modified stiffness matrix is given by the Sherman-Morrison formula as:

$$\underline{K}_m^{-1} = \underline{K}^{-1} - \frac{\underline{K}^{-1}\,\underline{u}\,\underline{u}^T\,\underline{K}^{-1}}{1/G + \underline{u}^T\,\underline{K}^{-1}\,\underline{u}} \tag{2.3.19}$$

It follows that adding stiffness G to the structure in the direction $\underline{u}$ is equivalent to subtracting flexibility H in the direction $\underline{U}$, where:

$$H = \frac{1}{1/G + \underline{u}^T\,\underline{K}^{-1}\,\underline{u}} \tag{2.3.20}$$

and

$$\underline{U} = \underline{K}^{-1}\,\underline{u} \tag{2.3.21}$$

2.3.7 Modified Equilibrium Equations

When a rank one stiffness modification, $G\,\underline{u}\,\underline{u}^T$, is made, the solution of the modified equilibrium equations can be found as follows.

(1) Perform two back substitutions with the unmodified $\underline{K}$:

$$\underline{v}_1 = \underline{K}^{-1}\Delta\underline{R} \tag{2.3.22}$$

$$\underline{v}_2 = \underline{K}^{-1}\,\underline{u} \tag{2.3.23}$$

(2) Perform two dot products:

$$\beta = \underline{u}^T\,\underline{v}_1 \tag{2.3.24}$$

$$\gamma = \underline{u}^T\,\underline{v}_2 \tag{2.3.25}$$

(3) Form $\Delta \underline{r}$ from:

$$\Delta \underline{r} = \underline{v}_1 - \left(\frac{\beta}{1/G + \gamma} \right) \underline{v}_2 \tag{2.3.26}$$

2.3.8 Higher Order Modification

Higher order modifications to a stiffness matrix can be made by a series of rank one modifications. Calculation of the displacement increment can then be done with successive use of the Sherman-Morrison formula.

2.4 NEWTON-RAPHSON ITERATION

2.4.1 General

The Newton-Raphson (NR) iteration scheme is well known as a method for the analysis of nonlinear structures. It is reviewed here as a basic solution scheme to introduce the concepts and operations found in the more general methods to be discussed later.

2.4.2 Algorithm

If the current state is an equilibrium state, the iterative sequence for NR iteration is as follows (Fig. 2.1).

(1) The tangent stiffness is formed in the current state.

(2) A load increment is added to the structure. A displacement increment is found by solving the equilibrium equations.

(3) A state determination is carried out, and the structure resisting force is calculated.

(4) The unbalanced force is calculated. Convergence is checked. If converged, go to Step 1. If not converged, continue.

(5) The tangent stiffness is formed in the new current state.

(6) The displacement increment due to the unbalanced load is calculated.

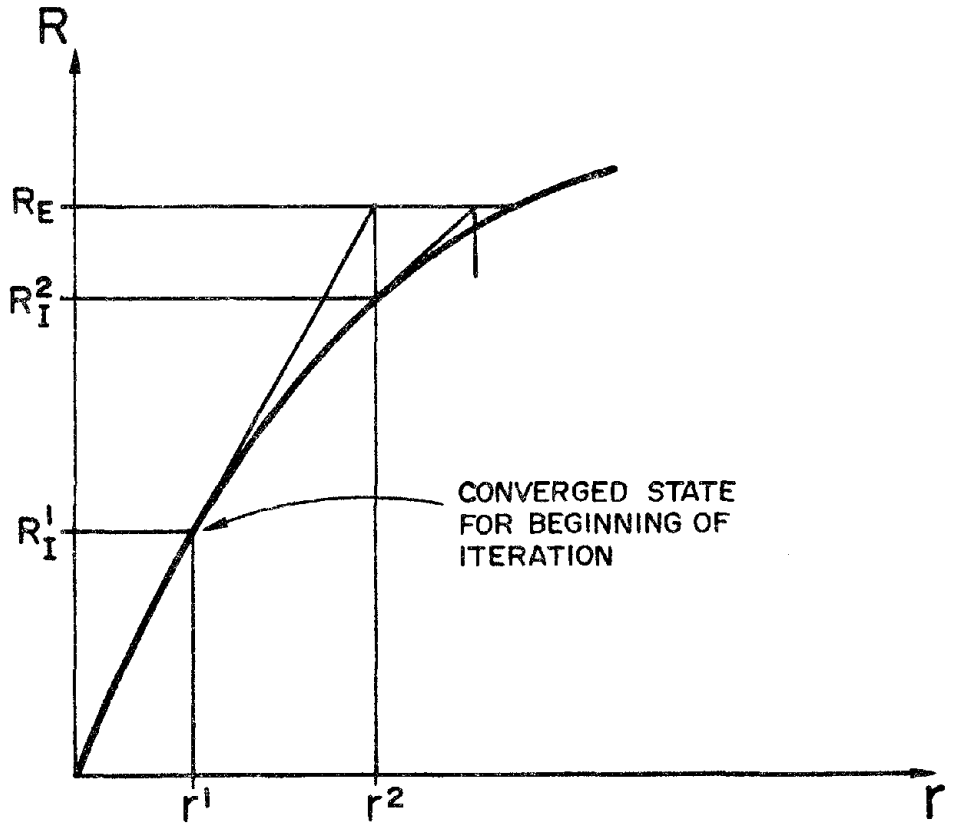

Fig. 2.1. Newton-Ralphson iteration.

(7) A state determination is carried out, and the structure resisting force is formed.

(8) The unbalanced force is calculated. Convergence is checked. If converged, go to Step 1. If not converged, go to Step 5.

2.4.3 Phases

There are two phases in the above sequence. In the *advancing phase* (Steps 1-4), a load increment is applied. In the *correcting phase* (Steps 4-8), the load magnitude is kept constant and the solution iterates in search of a converged state.

2.4.4 Tasks

The solution scheme can be separated into four distinct tasks, as follows, each of which is performed in both phases.

(a) *Linearization.* The tangent stiffness is formed in the current state.

(b) *Displacement Prediction.* A displacement increment is obtained by solving the equilibrium equations.

(c) *State Determination.* The element deformations, element actions, element resisting forces, and structure resisting forces are calculated.

(d) *Convergence Check.* The external force vector is formed, and the unbalanced force vector is checked for convergence.

2.4.5 Weaknesses

The basic NR scheme, although effective in many cases, is not necessarily the most economical solution scheme and does not always provide rapid or reliable convergence. Some weaknesses of the method are as follows.

(a) Linearization Expense.

The computation involved in linearization and equation solving may be large. When the solution is nearly converged, only small changes will take place in the tangent stiffness, and a new linearization may not be needed.

(b) Load Increments.

The size of each load increment must be predetermined. Because the structure stiffness varies throughout the analysis, equal load increments will produce unequal displacement increments and unequal unbalanced forces. A load increment that produces reasonable displacements with fast convergence initially may predict large displacement increments with slow or nonexistent convergence as the structure yields and becomes flexible. A substantial amount of trial and error may be needed to determine appropriate load increments.

(c) Step Direction.

The analyst will usually have no alternative but to specify positive load increments. In many structures the strength can reach a maximum and then decrease. In order to follow an equilibrium path in such cases, negative load increments must be applied.

(d) Constant Load Iteration.

The load is kept constant during the iteration. If the structure strength reaches a maximum then decreases, it is possible for the applied load to be greater than the structure strength, in which case convergence is impossible (at least near the predicted displaced state).

(e) Sudden Nonlinearities.

In some problems distinct "events" occur that drastically alter the stiffness (for example, gap closure). If such an event occurs in either the advancing or correcting phase, the calculated displacement increment may be a poor estimate of the actual increment and result in a large unbalance.

(f) Nonconvergence.

If the analysis does not converge in a specified number of iterations, it is necessary either to quit or to continue from a nonconverged state.

(g) Path Dependent State Determination.

The structure state is updated at each iteration, and nonconverged states are thus used as reference states. Because of the path dependent behavior of inelastic materials, use of nonconverged reference states may cause the calculated material response to differ from the true response.

2.4.6 Variations on NR

To overcome some of the weaknesses of NR iteration, a number of modifications of the basic scheme have been proposed. These modifications can be classified into four categories, as follows:

(a) Variations in the stiffness formation.

(b) Variations in the advancing phase.

(c) Variations in the correcting phase.

(d) Special logic in case of large unbalance or nonconvergence.

The modifications are discussed in the following sections.

2.5 VARIATIONS IN STIFFNESS FORMATION

2.5.1 Modified Newton Methods

If the analysis is not highly nonlinear, the structure stiffness will not change much between iterations. It is then possible to use the existing stiffness matrix to predict the displacement increment. Displacements predicted in this way may not converge as fast as those predicted with an updated stiffness, but because each iteration is cheaper, more iterations can be performed for the same cost. Variations on the NR scheme that do not reformulate the stiffness every iteration are commonly termed *modified Newton* methods. Some of these are described below and illustrated in Fig. 2.2.

In *initial stiffness* iteration, the initial stiffness matrix, $\underline{K}_o$, is used for all the displacement increment calculations. In *constant stiffness* iteration the stiffness matrix is updated for the

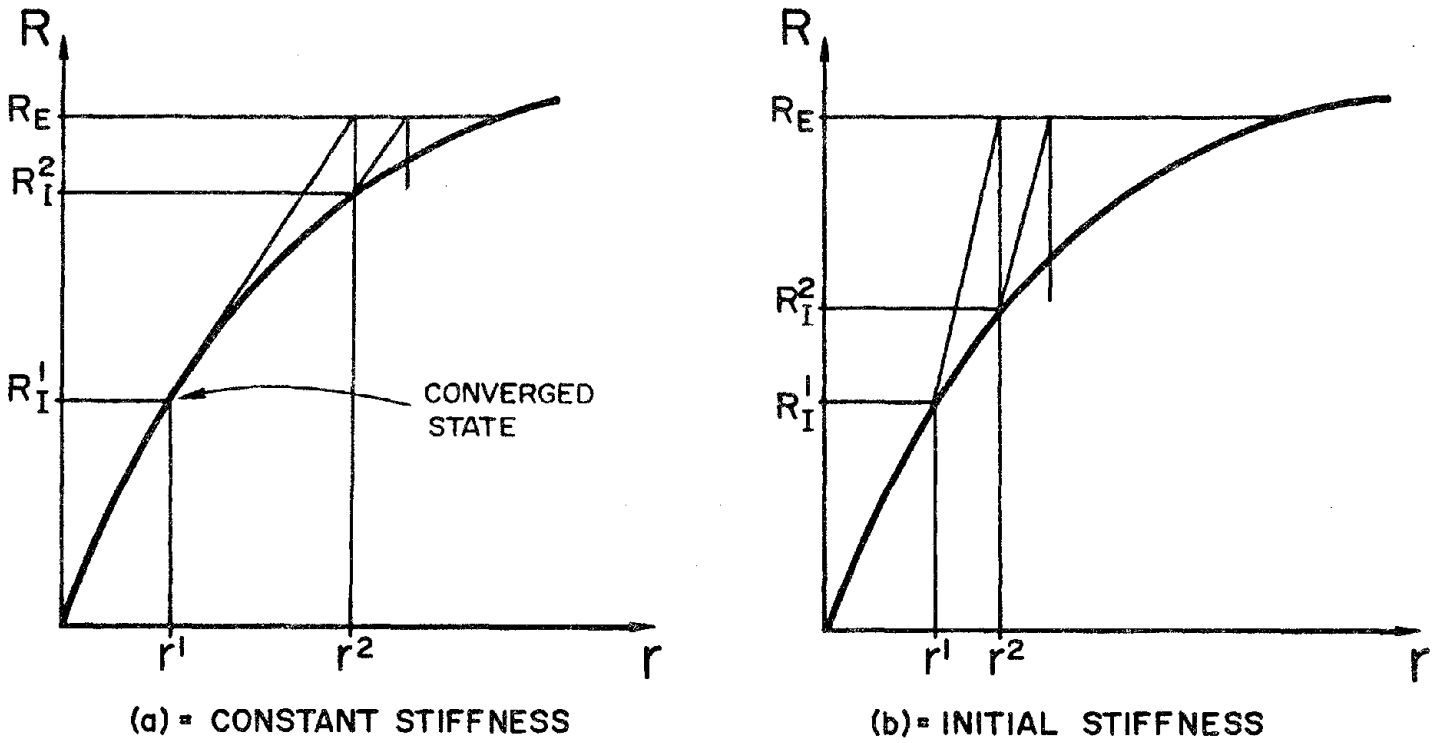

Fig. 2.2. Modified Newton methods.

advancing phase and then kept constant during the iteration. Other variations include (a) updating the stiffness in the advancing phase and at specified iterations during the correction phase and (b) reforming the stiffness when necessary according to some specified criteria (e.g., keep $\underline{K}$ constant after the solution converges to some preliminary tolerance).

2.5.2 Quasi-Newton Methods

2.5.2.1 General

An alternative to reformulating the stiffness every iteration or keeping the stiffness constant is modifying the stiffness in some way. This is the idea behind "quasi-Newton" methods. Modifications are typically done so that the following guidelines are met [3].

(a) The modified stiffness matrix, $\underline{K}_m$, for any iteration, i, should be a secant stiffness matrix for the displacements calculated in the previous iteration, $\Delta \underline{r}^{i-1}$. That is,

$$\underline{K}_m \Delta \underline{r}^{i-1} = \underline{R}^i_I - \underline{R}^{i-1}_I = \underline{R}^{i-1}_U - \underline{R}^i_U \tag{2.5.1}$$

where $\underline{R}^i_I$ and $\underline{R}^i_U$ are the resisting and unbalanced forces, respectively, at the beginning of iteration i.

(b) If $\underline{K}$ is symmetric and positive definite, $\underline{K}_m$ should also be symmetric and positive definite.

(c) Displacement increments using $\underline{K}_m$ should be cheap to calculate.

There is extensive literature on methods of stiffness modification, most of it from fields other than structural analysis. The mathematical formulation and convergence properties of some of these methods have been explored in [3], [4], and [5]. The method which has received the most attention for structural analysis is the Broyden-Fletcher-Goldfarb-Shanno (BFGS) method [3]. It proposes modification of the stiffness matrix by addition of a rank two matrix. Consider first, however, the quasi-Newton method based on a rank one modification.

2.5.2.2 Rank One Stiffness Correction

The simplest stiffness modification is the addition of a rank one matrix. There is only one rank one correction which satisfies Eqn. (2.5.1), given by:

$$\underline{K}_m = \underline{K} + \underline{K}_u \tag{2.5.2}$$

where

$$\underline{K}_u = -\underline{R}_U^i (\underline{R}_U^i)^T / (\underline{R}_U^i)^T \Delta \underline{r}^{i-1} \tag{2.5.3}$$

This correction changes the structure stiffness by an amount:

$$G = \frac{-(\underline{R}_U^i)^T \underline{R}_U^i}{(\underline{R}_U^i)^T \Delta \underline{r}^{i-1}} \tag{2.5.4}$$

in the direction of the current unbalanced force, $\underline{R}_U^i$.

The inverse of the modified stiffness, from Eqn. (2.3.19), is given by:

$$\underline{K}_m^{-1} = \frac{\underline{K}^{-1} \underline{R}_U^i (\underline{R}_U^i)^T \underline{K}^{-1}}{(\underline{R}_U^{i-1} - \underline{R}_U^i)^T (\underline{K}^{-1} \underline{R}_U^i)} \tag{2.5.5}$$

The calculation of the modified displacements, using Eqns. (2.3.22) through (2.3.26), requires only one back substitution.

A problem with the rank one update is that it may not be numerically stable, because the vectors in the denominator of Eqn. (2.5.5) may be orthogonal [5]. Higher order corrections have thus been developed, among them the rank two BFGS correction.

2.5.2.3 BFGS Correction

The BFGS modification is a rank two update, given by:

$$\underline{K}_B = \underline{K} + \frac{(\underline{R}_U^{i-1} - \underline{R}_U^i)\,(\underline{R}_U^{i-1} - \underline{R}_U^i)^T}{(\underline{R}_U^{i-1} - \underline{R}_U^i)^T \Delta \underline{r}^{i-1}} - \frac{\underline{R}_U^{i-1} (\underline{R}_U^{i-1})^T}{(\underline{R}_U^{i-1})^T \Delta \underline{r}^{i-1}} \tag{2.5.6}$$

The stiffness is changed in two directions. The first of these is given by the change in the resisting force for the last iteration. For constant load iteration, this can be calculated as the change in the unbalanced force, namely:

$$\underline{u}_1 = \frac{(\underline{R}_U^{i-1} - \underline{R}_U^i)}{((\underline{R}_U^{i-1} - \underline{R}_U^i)^T (\underline{R}_U^{i-1} - \underline{R}_U^i))^{1/2}} \tag{2.5.7}$$

A stiffness of magnitude

$$G_1 = \frac{(\underline{R}\bar{U}^{-1} - \underline{R}\bar{U})^T(\underline{R}\bar{U}^{-1} - \underline{R}\bar{U})}{(\underline{R}\bar{U}^{-1} - \underline{R}\bar{U})^T \Delta \underline{r}^{i-1}} \tag{2.5.8}$$

is added in this direction.

The second stiffness change is in the direction of the unbalanced force for the previous iteration, namely:

$$\underline{u}_2 = \frac{\underline{R}\bar{U}^{-1}}{((\underline{R}\bar{U}^{-1})^T \underline{R}\bar{U}^{-1})^{1/2}} \tag{2.5.9}$$

A stiffness of

$$G_2 = \frac{(\underline{R}\bar{U}^{-1})^T \underline{R}\bar{U}^{-1}}{(\underline{R}\bar{U}^{-1})^T \Delta \underline{r}^{i-1}} \tag{2.5.10}$$

is subtracted in this direction.

The inverse of the modified stiffness can be found by two applications of the Sherman-Morrison formula, as discussed by Dennis and More [5]. The calculation of the displacement increment (which involves only one back substitution) is presented in a convenient form by Matheis and Strang [3].

2.6 VARIATIONS IN THE ADVANCING STEP

2.6.1 Variable Load Magnitude

Strategies that automatically select the load increment during the analysis have been developed to avoid having to specify the load increments in advance. Three strategies are considered here. In the first two, the aim is to keep the unbalance at the end of the advancing phase constant in each step. In the third, the aim is to keep the number of iterations in each step constant.

2.6.1.1 Bergan's Current Stiffness Parameter

Bergan [2] uses the current stiffness parameter, S_p, as a guiding quantity for selecting the load increment. If the linearization is regarded as a first order Taylor series approximation, the

truncation error in the advancing phase varies with (a) the load increment and (b) a norm measure of the second derivative of r with respect to the load magnitude. That is,

$$\tau = c\alpha|\ddot{r}| \tag{2.6.1}$$

where

τ = truncation error;

c = a constant;

α = load increment; and

$|\ddot{r}|$ = norm of the second derivative of r with respect to the load magnitude.

Because S_p is an approximation to the first derivative of the displacements (with respect to the load magnitude), the change in S_p divided by the change in load magnitude is an approximation to the second derivative. Hence, the truncation error will be approximately constant in each step if the load increment is chosen so that the change in S_p is constant. The choice of the load increment is given by:

$$\alpha^i = \alpha^{i-1} c/\Delta S_p^{i-1} \tag{2.6.2}$$

where

c = a constant;

α^i = load increment in Step i; and

ΔS_p^{i-1} = change in S_p in Step i-1.

This method of step selection will result in small steps where the solution is strongly non-linear and large steps in nearly linear regions.

2.6.1.2 Scaling Based on Unbalance

Step scaling based on the unbalance can be done directly if a state determination is added to the process. In this method, a trial step is taken, a state determination is performed, and the unbalance is calculated. If the unbalance is too large (based on some tolerance), the step is repeated with a reduced load increment.

The procedure is as follows:

(1) Select a load increment (based on any method).

(2) Solve for the displacement increment.

(3) Perform a state determination.

(4) Calculate the unbalance.

(5) If the unbalance is greater than the specified tolerance, scale the load increment according to:

$$\alpha_s = \alpha U/|R_U| \tag{2.6.3}$$

where

α_s = scaled load increment;

U = unbalance tolerance; and

$|R_U|$ = norm of unbalanced load.

and repeat from Step (3). If the unbalance is less than the tolerance, enter the correcting phase.

Because the unbalance varies approximately quadratically with the displacement increment (based on the truncation error), a linear scaling will, in most cases, be sufficient to reduce the unbalance below the allowable tolerance.

2.6.1.3 Scaling Based on Number of Iterations

A method in which the load increment is adjusted based on the number of iterations has been suggested by Crisfield [6]. The magnitude of each step (except the first, which must be specified) is calculated as follows for Step i:

$$\alpha^i = \alpha^{i-1} I/I^{i-1}$$

where

I^{i-1} = number of iterations for convergence in Step i-1;

α^{i-1} = load increment for Step i-1; and

I = desired number of iterations for convergence.

If a large number of iterations was required for convergence in the preceding step, the load increment for the current step will be smaller.

2.6.2 Displacement Control

2.6.2.1 General

An alternative to varying the load increment is to control the size of the displacement increment directly. Various measures of the displacement increment can be controlled, including (a) a single displacement degree of freedom, (b) a scalar displacement, and (c) the "arc length" of the increment.

2.6.2.2 Single Degree of Freedom Control

Haisler and Stricklin [7] describe a step-by-step method without iteration in which a selected displacement (the *controlled displacement)* is increased by a specified amount in each step. In any step, the unbalanced load at the beginning of the step, plus some load increment, is applied. The magnitude of the load increment is initially unknown and is chosen to increase the controlled displacement by the specified amount. Because the displacement vector for any step consists of two parts, one due to the load increment and one due to the unbalance, the displacements cannot simply be scaled linearly to meet the displacement constraint, and a special computational procedure is needed to determine the required load magnitude.

The computational procedure proposed by Haisler and Stricklin involved partitioning of the equilibrium equations as follows.

(1) The equilibrium equations are set up keeping the applied load vector and the unbalanced force vector separate:

$$\underline{K}\,\Delta\underline{r} = \alpha_E \Delta\underline{R}_E + \underline{R}_U \tag{2.6.4}$$

where

α_E = unknown load increment magnitude; and

$\Delta \underline{R}_E$ = external load vector corresponding to a unit load magnitude.

(2) The equilibrium equations are partitioned to separate out the controlled displacement:

$$\begin{bmatrix} \underline{K}_{11} \underline{R}_E^1 \\ \underline{K}_{21} \underline{R}_E^2 \end{bmatrix} \begin{bmatrix} \Delta \underline{r}^1 \\ \alpha_E \end{bmatrix} = \begin{bmatrix} \underline{R}_U^1 \\ \underline{R}_U^2 \end{bmatrix} - \Delta \bar{r} \begin{bmatrix} \underline{K}_{12} \\ \underline{K}_{22} \end{bmatrix} \tag{2.6.5}$$

where

$\underline{K}_{11}$ = stiffness matrix with row i and column i removed;

$\underline{K}_{12}$ = column i of $\underline{K}$ without element i;

$\underline{K}_{21}$ = row i of K without element i;

$\underline{K}_{22}$ = element $\underline{K}_{ii}$;

$( )^1$ = vector without element i;

$( )^2$ = element i of vector; and

$\Delta \bar{r}$ = controlled displacement increment.

(3) The partitioned equations are solved for α_E:

$$\alpha_E = (\underline{R}_U^2 - \underline{K}_{21} \underline{A} - \underline{K}_{22} \Delta \underline{r}^1)/(\underline{K}_{21} \underline{B} - \underline{R}_E^2) \tag{2.6.6}$$

where

$\underline{A}$ is given by $\underline{K}_{11} \underline{A} = \underline{R}_U^1 - \Delta \bar{r} \, K_{12}$; and

$\underline{B}$ is given by $\underline{K}_{11} \underline{B} = \underline{R}_E^1$.

Batoz and Dhatt [8] recognized that the Haisler-Stricklin scheme is computationally cumbersome and suggested a simpler scheme. The steps are as follows:

(1) Solve the equilibrium equations separately for the displacements due to the unbalanced force and an applied load of arbitrary magnitude:

$$\underline{K} \, \Delta \underline{r}_U = \Delta \underline{R}_U \tag{2.6.7}$$

$$\underline{K} \, \Delta \underline{r}_E = \Delta \underline{R}_E \tag{2.6.8}$$

(2) Combine the two displacement vectors so that the controlled displacement has the required magnitude:

$$\Delta\bar{r} = (\Delta\underline{r}_U)_i + \alpha_E (\Delta\underline{r}_E)_i \tag{2.6.9}$$

where

$( )_i$ = element i of vector; and

$\Delta\bar{r}$ = controlled displacement increment.

Hence,

$$\alpha_E = (\Delta\bar{r} - (\Delta\underline{r}_U)_i)/(\Delta\underline{r}_E)_i \tag{2.6.10}$$

2.6.2.3 Scalar Displacement Steps

The method described in the preceding section has been presented in a generalized form by Powell and Simons [9]. In this method, the controlled displacement is not limited to a single d.o.f. but is a scalar displacement characterized by a unit vector, $\underline{b}$. That is,

$$\Delta\bar{r} = \underline{b}^T\Delta\underline{r} = \underline{b}^T\Delta\underline{r}_U + \alpha_E \underline{b}^T\Delta\underline{r}_E \tag{2.6.11}$$

and hence,

$$\alpha_E = (\Delta\bar{r} - \underline{b}^T\Delta\underline{r}_U)/\underline{b}^T\Delta\underline{r}_E \tag{2.6.12}$$

2.6.2.4 Arc Length Steps

An alternative method of advancing the solution has been proposed by Riks [10] and Crisfield [6] and discussed by Ramm [11]. The "arc length" of the step, s, is defined by the Euclidean norm of a vector containing both the load increment and the displacement increment. That is,

$$s = (\alpha_E^2 + \Delta\underline{r}^T\Delta\underline{r})^{1/2} \tag{2.6.13}$$

in which

$\Delta\underline{r}$ = displacement increment; and

α_E = load increment.

The arc length, in geometrical terms, is an approximation to the length of the step in displacement-load space. Choice of the load increment based on a specified arc length, $\bar{s}$, gives a load increment of:

$$\alpha_E = \frac{(-\Delta \underline{r}_U^T \Delta \underline{r}_E \pm ((\Delta \underline{r}_U^T \Delta \underline{r}_E)^2 - (1 + \Delta \underline{r}_E^T \Delta \underline{r}_E)(\Delta \underline{r}_U^T \Delta \underline{r}_U - \bar{s}^2))^{1/2})}{(1 + \Delta \underline{r}_E^T \Delta \underline{r}_E)} \qquad (2.6.14)$$

Because both the load increment and the displacement increment are included in s, the size of the load increment should be determined primarily by the largest of these quantities. When the structure is very flexible, the displacements will control the size, and when the structure is very stiff, the load increment will control.

This choice of load increment has some weaknesses, however. First, the quantities that make up the arc length do not have the same units (one is a load term and the rest are displacements). Because of this, the relative influence of the load term depends on the units which are chosen. Second, since the arc length contains all degrees of freedom, local nonlinearities tend to get lost or diluted, especially in analyses with many d.o.f.

From numerical experience, Crisfield recommends that the load increment not be included in the calculation [6]. The resulting strategy is then a displacement controlled method using the Euclidean norm of the displacement increment to control the size of the load increment. The resulting load increment is given by:

$$\alpha_E = (-\Delta \underline{r}_E^T \Delta \underline{r}_U \pm ((\Delta \underline{r}_E^T \Delta \underline{r}_U)^2 - (\Delta \underline{r}_E^T \Delta \underline{r}_E)(\Delta \underline{r}_U^T \Delta \underline{r}_U))^{1/2})/(\Delta \underline{r}_E^T \Delta \underline{r}_E) \qquad (2.6.15)$$

2.6.3 Choice of Step Direction

2.6.3.1 General

In an analysis of a buckling structure, the load magnitude may increase up to a maximum value and then decrease as the structure continues to deform. This will be termed *load reversal*. For load stepping, negative load increments must be applied to follow the equilibrium path.

For displacement stepping a similar problem may occur, in which the controlled displacement reaches a maximum and then decreases. To continue the analysis past this *displacement*

reversal, it is necessary to change from positive to negative displacement steps.

In general, if the controlled quantity (load or displacement) does not experience a reversal, then the *step direction* can simply be chosen as positive in each step. If, however, the controlled quantity is subject to reversals, a procedure to choose the step direction is necessary. Two procedures are considered here, the first based on the current stiffness parameter and the second on continuity of the displacement increment.

2.6.3.2 Current Stiffness Parameter

The current stiffness parameter, S_p, is used by Bergan to choose both the load magnitude and the step direction. When the load magnitude changes direction, S_p passes through zero. Monitoring S_p enables the direction of the load increment to be chosen as follows:

(1) Initially S_p is equal to one and the load increment is positive.

(2) As the stiffness decreases, S_p becomes smaller. As long as S_p remains positive, the load increment is chosen as positive.

(3) If S_p passes through zero and changes sign, the load increment is chosen negative, until S_p passes back through zero and becomes positive.

One problem with this technique is that S_p also changes sign when the scalar displacement r_s, given by:

$$r_s = \Delta \underline{r}^T \Delta \underline{R} \tag{2.6.16}$$

changes sign for a positive $\Delta \underline{R}$. In this case, S_p passes through infinity and the step direction should remain the same. In problems with strong local nonlinearities, it may not always be obvious whether S_p changes sign by passing through zero or infinity. This uncertainty could cause confusion in the step direction choice.

2.6.3.3 Continuity

Crisfield [6] describes a method for choosing the step direction based on continuity of the displacement increments. The idea is that in order to keep the solution from going back on

itself, consecutive displacement increments should be in the same direction. In each step, the displacement direction that makes the smaller angle with the previous displacement increment is the one chosen.

2.7 VARIATIONS IN THE CORRECTING PHASE

2.7.1 General

As discussed in Section 2.4.6, the basic NR scheme has some weaknesses in the correcting phase. First, iteration is done with constant load, which may lead to divergence. Second, path dependent state determination is used, which can introduce errors in the material response. Third, no provision exists to alter the magnitude or direction of the displacement increment found by solving the equilibrium equations.

Several variations in the correcting phase have been proposed to overcome some of these weaknesses. Instead of iterating with constant load, schemes have been devised that (a) do not iterate, (b) do not iterate when the stiffness is very low, and (c) iterate with constant displacement. Schemes that alter the magnitude and/or direction of the calculated displacement increment in order to aid convergence have also been developed, among them *line search* methods to select the magnitude and the *conjugate Newton* method to alter the direction. Also, instead of path dependent state determination, path independent state determination is easily incorporated. These variations are discussed below.

2.7.2 No Iteration

The scheme proposed by Haisler and Stricklin (described in Section 2.6.2.1) is performed without iteration. The solution is carried out step-by-step with the unbalanced load and an increment of load applied each step.

Bergan suggests that iteration be suspended only near critical points [2]. He recommends the use of the current stiffness parameter, S_p, as a criterion for iteration, and iterations are performed only if S_p is not near zero. Because S_p is a measure of the scalar stiffness of the struc-

ture, it will be close to zero when the structure stiffness is very low.

In schemes with iteration, it is usual to iterate until the unbalanced load is small so that only small unbalances are carried over from step to step. In schemes that do not iterate, substantial unbalances can be carried over, which may contribute to a drift from the equilibrium path.

2.7.3 Displacement Control

Methods that control displacements can be used in the correcting phase, and can be thought of as a special case of the method described in Section 2.6.2.2 for controlling the displacement increment in the advancing phase. In the correcting phase, iteration with some scalar displacement held constant corresponds to specifying the increment in that scalar displacement to be zero. Using the notation introduced in Section 2.6.2.2, iteration is done with:

$$\Delta\tilde{r} = \underline{b}^T \Delta\underline{r} = 0 \tag{2.7.1}$$

The load increment is calculated by:

$$\alpha_E = -\underline{b}^T \Delta\underline{r}_U / \underline{b}^T \Delta\underline{r}_E \tag{2.7.2}$$

An important point to note is that if iteration is done at a constant displacement, the load magnitude varies during the iteration. For a softening structure, the load magnitude will typically decrease during iteration. For structures that reach a maximum load, the problem of iterating at a load magnitude greater than the structure strength is overcome, because the load magnitude is automatically reduced during the iteration.

2.7.4 Path Independent State Determination

In NR iteration the state is updated in each iteration, so that the state determination is path dependent. It has been noted [2,12] that path dependent state determination can lead to significant errors if the path followed is far from the equilibrium path. An alternative scheme is to use path independent state determination.

If the element strains increase progressively during the iteration sequence, there will usually be little difference between the final states calculated by the two schemes. However, if the

strains increase in early iterations and then decrease, the path dependent scheme may incorrectly unload yielded elements. Consider, for example, an analysis of a softening structure with constant displacement iteration. The load magnitude typically decreases in the iteration phase, although the accumulated load increment from the beginning of the step is positive. A path dependent scheme will allow unloading of yielded elements as the load decreases, whereas a path independent scheme will not because it is based on the accumulated displacement increment. Path independent state determination is thus recommended for displacement controlled analysis.

2.7.5 Line Search

The displacement increment found by solving the equilibrium equations does not necessarily give the best estimate of the equilibrium state. Instead, some multiple of the displacement increment, $\beta\Delta\underline{r}$, may be better. In line search methods, β is chosen to minimize some measure of the unbalance, usually by a successive trial procedure (i.e., successive values of β are chosen based on previous values of β and $\underline{R}_U$, until $\underline{R}_U$ is reduced below some tolerance).

Matheis and Strang [3] discuss the use of a line search routine in conjunction with the BFGS stiffness modification scheme. The technique can be used with any method for choosing the displacement increment.

The line search is carried out as follows:

Phase 1: Upper and lower values of β are sought which bound a zero value of unbalance.

(1) A measure of the unbalance at the beginning of the step ($\beta = 0$) and for the calculated displacement increment ($\beta = 1$) is calculated as follows:

$$R_n(\beta) = \Delta\underline{r}^T \underline{R}_U(\beta) \tag{2.7.3}$$

$R_n(\beta)$ is a measure of the external work that the unbalanced load does on the structure.

(2) If the unbalance measures are of opposite signs, the zero is bounded, so enter Phase 2. If the unbalance measures are of the same sign, then double the step size (β = 2) and recalculate the unbalance, this time using $\beta_1 = 1$ and $\beta_2 = 2$ as the bounds. Repeat this step until a zero unbalance is bounded or until a maximum number of trials have been performed.

Phase 2: Find β to minimize the unbalance.

(1) Based on the values of β_1 and β_2 and the corresponding unbalances, use a linear approximation to choose the new value of β. That is,

$$\beta = \beta_1 + \frac{R_n(\beta_1)\,(\beta_2 - \beta_1)}{R_n(\beta_2) - R_n(\beta_1)} \tag{2.7.4}$$

(2) Evaluate the unbalance for this value. If it is less than a specified tolerance, then quit. If not, choose the two points with opposite signed unbalances and repeat Step (1). Convergence is obtained when the unbalance is less than a specified proportion of $R_n(0)$. That is, when:

$$R_n(\beta) < c\, R_n(0). \tag{2.7.5}$$

where c = a constant between zero and one.

2.7.6 Conjugate Newton

In all of the methods described so far, the search direction has been calculated by solving the equilibrium equations. Irons [13] has introduced the conjugate Newton method in which the direction of the displacement increment is modified in each iteration. It is based on an idea from the conjugate gradient method (an iterative method of solving function minimization problems), in which a set of search directions that are conjugate result in an efficient search.

In the conjugate Newton method, a displacement, increment $\Delta \underline{r}^i$, is first calculated as in constant stiffness iteration. This increment is then modified so that it is *conjugate* to all previous directions, i.e., so that:

$$(\Delta \underline{r}^i)^T \underline{K}\, \Delta \underline{r}^j = 0 \qquad j = i-1,\ i-2,\ldots,\ 1 \tag{2.7.6}$$

The steps are as follows for each iteration.

(1) An unmodified displacement increment is obtained as:

$$\Delta \underline{r}^i = \underline{K}^{-1} \Delta \underline{R}^i \tag{2.7.7}$$

(2) The displacement increment is made conjugate to the previous displacement increments.

$$\Delta \underline{r}_c^i = \Delta \underline{r}^i - c \Delta \underline{r}_c^{i-1} \tag{2.7.8}$$

where c is a constant determined so that:

$$(\Delta \underline{r}_c^i)^T \underline{K} \Delta \underline{r}_c^{i-1} = 0 \tag{2.7.9}$$

where

$\Delta \underline{r}_c^i$ = modified displacement increment; and

$\Delta \underline{r}_c^{i-1}$ = displacement increment from previous iteration.

(3) The magnitude of the step is calculated by a line search.

2.8 SPECIAL LOGIC

2.8.1 General

In cases where large unbalances develop or the iteration fails to converge, it may be necessary to implement special logic in order to complete the analysis. Three schemes which have been developed to deal with these problems are considered in this section.

The first scheme aims to reduce the unbalance by making it orthogonal to the external force. The second scheme is applicable to problems in which a large unbalance may result from distinct "events" (e.g. gap closure). A strategy that advances the solution from event to event is described. Finally, for problems that do not converge within a specified number of iterations, a scheme to restart the analysis from the last converged state is described, along with a procedure to predict convergence.

2.8.2 Orthogonalized Unbalance

Bergan [1] has introduced a method in which the unbalanced force is orthogonalized with respect to the applied force in each iteration. The method is as follows for each iteration.

(1) Calculate the displacement increment in the usual way.

(2) Perform the state determination and calculation of the resisting force, $\underline{R}_I$.

(3) Calculate the unbalanced force, $\underline{R}_U$.

(4) Calculate the component of $\underline{R}_U$ parallel to $\underline{R}_E$ and subtract it from both $\underline{R}_U$ and $\underline{R}_E$. That is:

$$\underline{R}_{EO} = (1.0 - \gamma)\,\underline{R}_E \tag{2.8.1}$$

$$\underline{R}_{UO} = \underline{R}_U - \gamma\,\underline{R}_E \tag{2.8.2}$$

where

$$\gamma = (\underline{R}_U^T \underline{R}_E)/(\underline{R}_E^T \underline{R}_E)$$

and the subscript O represents an orthogonalized vector.

(5) Check convergence. If converged, go to the next step. If not, repeat from Step (1).

2.8.3 Event-to-Event Strategy

A strategy for problems that are linear (or nearly linear) between well defined *events* is to advance the solution from event to event, rather than take specified steps with iteration. This strategy is discussed, for example, by Porter [14]. The purpose of the event-to-event strategy is to follow the equilibrium path closely at all times by updating the stiffness and state each time an event occurs. In this way the unbalance will, ideally, never get large.

The procedure is as follows, assuming the current state is an equilibrium state.

(1) Linearize about the current state.

(2) Calculate the displacements for an arbitrary load increment.

(3) Predict the next event. Events typically correspond to changes of state in the elements, and event prediction calculations must be performed for each element to determine whether the calculated displacement increment will cause an event. If an event is predicted, a scale factor is determined that will bring the solution just to the predicted event.

(4) Scale the displacement increment and add it to the current displacements. Update the structure state. Continue from Step (1).

Although the event-to-event strategy is presented above as a solution scheme in itself, it is possible to incorporate the concept of event prediction into solution strategies of NR type.

2.8.4 Restepping If No Convergence

Despite the wide range of strategies available, convergence will not always occur. In such cases, it is helpful to have a *restepping* capability available. This means that if convergence is not obtained at the end of the correcting phase, the step size is reduced and the step is taken again from the backup state. Hibbitt has incorporated this type of procedure into the program ABAQUS [15].

2.8.5 Convergence Prediction

The restepping option can be augmented by the use of a routine that predicts convergence. After a specified number of iterations, a prediction is made as to whether the solution will converge in the allowable number of iterations. If nonconvergence is predicted, the restepping option is exercised. This type of prediction can save doing futile iterations.

2.9 DESIRABLE FEATURES OF A GENERAL ALGORITHM

It is possible to construct a general solution algorithm which retains the structure of NR iteration but which incorporates most of the variations described in the preceding sections. The desirable features of such a general algorithm are those that overcome the difficulties of standard NR iteration. Some of these features are as follows.

(1) Strategies that avoid the high cost of linearization:

   (a) Modified Newton methods, such as initial stiffness and constant stiffness iteration, that keep the stiffness constant for a number of iterations.

(b) Quasi-Newton methods, such as BFGS, that use simple modifications to the stiffness matrix.

(2) Aids to convergence, especially for buckling and snap-through problems:

(a) Variable step size based on the current stiffness parameter or the unbalanced load.

(b) Line search in the correcting phase to minimize the unbalance.

(c) Direct control over the displacement increment.

(d) Iteration with constant displacement.

(e) Choice of path dependent or path independent state determination.

(3) Special strategies, such as event-to-event, to deal with particular types of behavior.

(4) Restepping capabilities combined with iteration prediction to deal with nonconvergence.

An algorithm which incorporates these features is described in the following chapter.

3. GENERAL SOLUTION SCHEME

3.1 TASKS AND PHASES

A general solution scheme for static nonlinear analysis has been and incorporated into the ANSR structural analysis program. The scheme is similar in structure to Newton-Raphson iteration. However, whereas there are four distinct tasks in NR iteration (linearization, calculation of new displaced state, state determination, and convergence check), there are only three in the general scheme. Linearization, which is performed every iteration in NR iteration, is not considered as a separate task but only as an option for selecting a new displaced state. The three tasks are:

(a) Selection of new displaced state.

(b) State determination.

(c) Calculation of unbalanced load and checking of convergence.

As in NR iteration, there are two phases in the analysis, namely, the advancing phase and the correcting phase. The three tasks are performed in each of the two phases.

3.2 STEPS AND SEGMENTS

An analysis is carried out in a series of *analysis steps*, each consisting of an *advancing phase* and a *correcting phase*. The *step size* (which defines the increments of load and displacement for the step) is controlled by a *stepping parameter*, which may be either a load or displacement quantity. For *load stepping*, the increment of load magnitude is controlled and the displacements follow. For *displacement stepping*, the increment in some displacement measure is controlled, and the load magnitude follows. The displacement measure will usually be a specified scalar displacement but may also be (a) a displacement norm or (b) the most critical of a set of specified scalar displacements. Details are presented later.

A complete analysis is divided into a number of *analysis segments*, each of which is divided into a number of analysis steps. The amount of load or displacement applied in a segment is

defined by the analyst in terms of a *segment value*, V_s. For load stepping, the segment value is the change in load magnitude for the segment. For displacement stepping, the segment value is the change in a scalar displacement or displacement norm. Within any analysis segment, stepping continues until the segment value is reached. The step size, S_s, is conveniently expressed as the product of a *step factor*, f_s (a number between 0 and 1), the segment value, and a direction factor, d_s (which is either 1 or -1). That is,

$$S_s = f_s V_s d_s \tag{3.2.1}$$

The step factor may be specified in advance by the analyst, or it may be determined automatically to satisfy certain criteria, depending on the stepping option selected by the analyst.

3.3 DISPLACEMENT INCREMENT

For the advancing phase, and in each iteration of the correcting phase, a displacement increment must be determined. The basic procedure for calculating a displacement increment is as follows.

(1) Displacements due to (a) the unbalanced load and (b) an arbitrary increment of applied load are calculated by solving the equilibrium equations:

$$\Delta \underline{r}_U = \underline{K}^{-1} \underline{R}_U \tag{3.3.1}$$

$$\Delta \underline{r}_E = \underline{K}^{-1} \Delta \underline{R}_E \tag{3.3.2}$$

(2) The displacement increment is formed as a linear combination of these two displacement vectors. That is,

$$\Delta \underline{r} = \Delta \underline{r}_U + \alpha_E \Delta \underline{r}_E \tag{3.3.3}$$

A constraint equation on either a load or displacement quantity is used to select the increment of applied load, α_E. For load stepping, α_E is specified directly, whereas for displacement stepping it is calculated using a displacement constraint equation.

3.4 ADVANCING PHASE

3.4.1 General

In the advancing phase, a load increment and a corresponding displacement increment must be determined to advance the solution along the equilibrium path. The nature of the step is determined by the stepping parameter, step factor, and direction factor.

3.4.2 Determination of the Step Factor

The step factor is a fraction of the complete segment value (load or displacement) to be applied in the step. For example, for load stepping a step factor of 0.5 means that half of the segment load is applied in the step. Similarly, for displacement stepping, a step factor of 0.5 means that half of the segment displacement is applied in the step.

The choice of step factor is an important consideration when advancing the solution. Too large a step may put the search far from the equilibrium path and may result in slow convergence or even divergence in the correcting phase. On the other hand, too small a step may be expensive because a large number of steps will be required to complete the analysis.

The step factor may remain constant throughout the analysis or it may vary. The available options are discussed in the following sections.

3.5 EQUAL STEPS

The analyst may specify that the solution is to be advanced in equal steps so that the stepping parameter is incremented by the same amount each step. For load stepping, the load increments will be equal and the displacement increments will generally be unequal. Conversely, for displacement stepping the displacement increments will be equal (as measured by the stepping parameter) and the load increments will generally be unequal.

The disadvantage of using equal steps is that the step factor must be chosen in advance by the analyst, and it may be difficult to select an appropriate step factor. It may be advantageous, therefore, to allow steps of variable size.

3.6 VARIABLE STEPS

3.6.1 General

Two options for variable steps are included in the scheme, namely (a) scaling based on unbalance and (b) scaling based on speed of convergence. Other options could be added; for example, scaling based on Bergan's current stiffness parameter.

3.6.2 Scaling Based on Unbalance

Scaling the step size based on the unbalance insures that some norm of the unbalance at the advancing phase never exceeds a specified value, U. An advancing step is taken using the current step factor, and the unbalance norm is calculated. If the unbalance exceeds U, the step size is reduced and the step is retaken. This procedure is repeated until the unbalance norm is below U.

Several scaling processes are possible. The one used in the ANSR implementation of the scheme is linear scaling, for which the step is scaled by:

$$S_s^i = S_s^{i-1} U / |\underline{R}_U^{i-1}| \tag{3.6.1}$$

where $(\ )^i = i^{th}$ trial and $|\underline{R}_U|$ is the unbalance norm.

This method results in smaller steps in regions of high nonlinearity. In the ANSR implementation, scaling in one step does not alter the step factor for the following step.

3.6.3 Scaling Based on Convergence Rate

In the event of nonconvergence in a step, an option exists to take the step again from the backup state, with a reduced step size. The assumption is that if convergence is slow (or if divergence occurs) the step size is too large and a smaller step would improve the chance of converging. The amount of the step reduction is specified by the analyst, by means of a reduction factor. For example, if the reduction factor is 0.25, the step factor is divided by four and the step is retaken.

If convergence is very rapid, the step size is assumed to be too small. The step factor is

then increased by a specified multiple if the solution converges in less than some minimum number of iterations. Use of this option will result in large steps where the solution is nearly linear.

For scaling based on convergence, the modified step factor is used in the following step.

3.7 STEPPING PARAMETER

3.7.1 General

The stepping parameter provides the analyst with the means of controlling the load and displacement increments. The best choice for the stepping parameter depends on the particular problem being solved.

3.7.2 Load Stepping

If the stepping parameter is the load magnitude (load stepping), then α_E in Eqn. (3.3.3) is equal to the step factor and is thus specified directly. The load increment for the advancing phase is the step factor times the segment value. Typically, for load stepping the load magnitude will be kept constant during the correcting phase, so that the load magnitude for the step is specified. However, it is possible for the load magnitude to vary during the correcting phase.

3.7.3 Displacement Stepping

3.7.3.1 General

For displacement stepping, the stepping parameter may be chosen as (a) a displacement norm, (b) a scalar displacement, or (c) the most critical of a set of scalar displacements. The quantity α_E in Eqn. (3.3.3) is then determined so that the stepping parameter is incremented an amount equal to the step size. The procedure is as follows.

3.7.3.2 Norms

If a displacement norm is chosen as the stepping parameter, α_E is chosen to make the norm of the displacement increment equal to the absolute value of the step size. For the

Euclidean norm this gives:

$$\alpha_E = (-\Delta \underline{r}_E^T \Delta \underline{r}_U \pm ((\Delta \underline{r}_E^T \Delta \underline{r}_U)^2 - (\Delta \underline{r}_E^T \Delta \underline{r}_E)(\Delta \underline{r}_U^T \Delta \underline{r}_U - S_s^2))^{1/2}) / \Delta \underline{r}_E^T \Delta \underline{r}_E \quad (3.7.1)$$

For the infinity norm (maximum absolute value):

$$\alpha_E = \min_i [(S_s - (\Delta \underline{r}_U)_i)/(\Delta \underline{r}_E)_i] \quad (3.7.2)$$

in which

$( )_i$ = i^{th} component of vector; and

$\min_i [\ ]$ = minimum absolute value for all i.

3.7.3.3 Scalar Displacements

If a scalar displacement is chosen as the stepping parameter, a stepping vector, $\underline{b}_s$, must be specified. In the advancing phase, the scalar displacement is incremented by an amount equal to the step size. Hence,

$$\alpha_E = (S_s - \underline{b}_s^T \Delta \underline{r}_U)/\underline{b}_s^T \Delta \underline{r}_E \quad (3.7.3)$$

Some possible choices for the stepping vector are as follows.

(a) Single degree of freedom: A single d.o.f., j, can be incremented by specifying $\underline{b}_s$ as a vector with all zero terms except term j=1.

(b) Strains: For an element such as a truss bar, displacement differences provide a measure of longitudinal strain. For example, consider a truss bar aligned parallel to the global X axis, with longitudinal degrees of freedom i and j. In this case, a stepping vector that measure the change in length is given by:

$$\underline{b}_s^T = [...0\ -1\ 0 \cdots 0\ +1\ 0...] \quad (3.7.4)$$

in which the -1 value is for d.o.f. i and +1 value for d.o.f. j.

(c) Rotations: Approximations to element rotations can be constructed as displacement differences. Consider, for example, a truss bar along the global X axis, and let degrees of freedom k and l be perpendicular to the bar. The rotation of the element, r_r, is given approximately by:

$$r_r = ((\Delta \underline{r})_l - (\Delta \underline{r})_k)/L \tag{3.7.5}$$

in which

$(\Delta \underline{r})_k$ is component k of $\Delta \underline{r}$;

$(\Delta \underline{r})_l$ is component l of $\Delta \underline{r}$; and

L is the element length.

The appropriate stepping vector is:

$$\underline{b}_s^T = [...0\ -1/L\ 0 \cdots 0\ +1/L\ 0...] \tag{3.7.6}$$

in which the term -1/L is for d.o.f. k and the term +1/L for d.o.f. l.

3.7.3.4 Critical Measure

In the above examples, the stepping parameter was a single predetermined quantity. In some analyses, there may be more than one quantity that needs to be controlled as the analysis progresses. The scheme implemented in ANSR allows up to four scalar displacements to be specified as possible stepping parameters. In any step, the stepping parameter actually used is the most critical of these possible choices.

The procedure is as follows:

(1) For each scalar displacement, a stepping vector, $\underline{b}_s$, and a corresponding segment value, V_s, are specified.

(2) In any step, the vector that is the most "sensitive" to the applied load is used as the critical vector. Sensitivity is determined by a variable C, given by:

$$C = \underline{b}_s^T \Delta \underline{r}_E / V_s \tag{3.7.7}$$

(3) The value of α_E follows from the critical vector as:

$$\alpha_E = (V_{sc} f_s d_s - \underline{b}_{sc}^T \Delta \underline{r}_U)/\underline{b}_{sc}^T \Delta \underline{r}_E \tag{3.7.8}$$

in which V_{sc} is the segment value for the critical vector, $\underline{b}_{sc}$.

The use of a critical vector to determine the step size has the advantage that the stepping parameter for the solution can change as the character of the solution changes. This feature

allows control of local nonlinearities (assuming they can be reflected in appropriate stepping vectors).

3.8 STEP DIRECTION

3.8.1 General

The calculation of the quantity α_E in Eqn. (3.3.3) has been discussed so far in terms of using the step size to scale the load or displacement increment. The sign of the step size, however, depends on the value of the direction factor, which is either one or minus one. The step direction must be chosen so that the solution advances in each step (i.e. so that the converged state found at the end of the step does not lie on a portion of the equilibrium path which has already been calculated). In general, the stepping parameter may reverse its direction during the analysis, and it is not always obvious in which direction the step should be taken.

Two methods are offered for the choice of the step direction. The first is to identify a *direction parameter* to guide the direction. If a direction parameter is specified, the step direction is chosen so that the value of the direction parameter increases in every step. The second method specifies the step direction indirectly, using Bergan's current stiffness parameter.

3.8.2 Direction Parameter

3.8.2.1 General

The best choice for the direction parameter in any analysis is not always known in advance. The essential requirement of a direction parameter is that it increase monotonically throughout the analysis. Even in the most complicated cases such parameters exist, but it may take trial and error by the analyst to find them. Possible choices include the load magnitude and various scalar displacements.

3.8.2.2 Load Magnitude

If the direction parameter is load magnitude, then the load is increased in every step (i.e.

α_E is always positive). This option will work only for structures that do not buckle and lose strength.

3.8.2.3 Critical Vector

If the critical measure option is used to determine the stepping parameter, then the critical vector may be used to determine the step direction. The direction is chosen so that the scalar displacement, r_s, defined by:

$$r_s = \underline{b}_{sc}^T \Delta \underline{r} \tag{3.8.1}$$

is positive.

3.8.2.4 New Vector

A new vector, different from the stepping vectors, can be specified as the *direction vector.* Any of the scalar displacements discussed as stepping parameters are possible direction parameters.

3.8.3 Bergan's Current Stiffness Parameter

The second method for direction choice is automatic load step selection by use of Bergan's current stiffness parameter. In this method, the sign of α_E starts out positive and changes sign each time S_p passes through zero. Although the step direction is not specified directly, its value is determined once the sign of α_E is specified.

3.9 CORRECTING PHASE

3.9.1 General

Following the advancing phase, one or more iterations are typically done to correct the solution in the region of the predicted state. Ideally, a converged state that is close to an equilibrium state (as measured by the unbalance) is found. However, specification of a large tolerance may allow significant unbalance.

Iterations are performed by holding an *iteration parameter* constant and adjusting the

remaining degrees of freedom. The iteration parameter may be either the load magnitude or a displacement quantity.

3.9.2 Constant Load Iteration

For constant load iteration, the iteration parameter is the load magnitude. Keeping the load magnitude constant is accomplished by setting $\alpha_E = 0$. The displacement increment is thus:

$$\Delta \underline{r} = \Delta \underline{r}_U \tag{3.9.1}$$

3.9.3 Constant Displacement Iteration

3.9.3.1 General

A displacement quantity can also be chosen as the iteration parameter. An iteration vector, $\underline{b}_I$, is specified to identify a scalar displacement (the *iteration parameter)* which is to be kept constant during the iteration. The magnitude of α_E is then chosen so that the change in the iteration parameter is zero. That is,

$$\alpha_E = -\underline{b}_I^T \Delta \underline{r}_U / \underline{b}_I^T \Delta \underline{r}_E \tag{3.9.2}$$

Some possible choices for the iteration parameter are discussed below. These include external work, arc length, and specified scalar displacements.

3.9.3.2 Constant Work Iteration

If a vector equal to the applied load vector is chosen as the iteration vector, then the quantity held constant during iteration is the external work (i.e. the applied load does no external work on the structure during the correcting phase). For this case:

$$\underline{b}_I = \underline{R}_E \tag{3.9.3}$$

3.9.3.3 Constant Arc Length

Iteration with constant "arc length" has been discussed by Riks [10] and Crisfield [6]. It is based on keeping the arc length, s, defined by:

$$s = (\alpha_E^2 + \Delta \underline{r}^T \Delta \underline{r})^{1/2} \tag{3.9.4}$$

constant throughout the iteration. This leads to a quadratic equation for α_E.

A simpler method is to iterate in such a way that the displacement increment for any iteration is normal to the accumulated displacement increment for the step, a method discussed by Ramm [11]. The iteration vector in this case is the displacement increment accumulated from the beginning of the current analysis step.

3.9.3.4 Constant Scalar Displacement

Several choices of scalar displacements for the advancing phase have been considered in Section 3.7.3.3. Any of these can be used for the correcting phase. For example, if the iteration vector is a unit vector with one in the j^{th} term and all other terms are zero, the j^{th} d.o.f. will be held constant during the iteration.

3.10 LINE SEARCH

An option is provided to carry out a line search on the magnitude of the displacement increment in each iteration of the correcting phase. The purpose of the line search is to choose the magnitude of the displacement increment that produces the smallest unbalance for the current external forces. The magnitude of the displacement increment is varied by multiplication by a scalar, β, while the external forces are held constant. The scaled displacement increment, $\Delta \underline{r}_s$, still satisfies the iteration condition, namely, that the iteration parameter remains constant during iteration.

The procedure is as follows:

(1) Perform state determination and unbalance calculations for three values of β, namely, $\beta = 1$, $\beta = \beta_u$, and $\beta = \beta_l$, where β_u and β_l are upper and lower limits specified by the analyst. Let the corresponding unbalance norms be $R_n(1)$, $R_n(\beta_u)$, and $R_n(\beta_l)$.

(2) A new value of β is predicted by choosing the value that corresponds to the minimum R_n, based on a parabolic approximation through the three known points. That is,

$$\beta = -\frac{A}{2B} \tag{3.10.1}$$

where

$$A = (\beta_1^2 R_n(\beta_2) + \beta_2^2 R_n(\beta_3) + \beta_3^2 R_n(\beta_1) - \beta_1^2 R_n(\beta_3) - \beta_2^2 R_n(\beta_1) - \beta_3^2 R_n(\beta_2))/D$$
$$B = (\beta_1 R_n(\beta_2) + \beta_2 R_n(\beta_3) + \beta_3 R_n(\beta_1) - \beta_1 R_n(\beta_3) - \beta_2 R_n(\beta_1) - \beta_3 R_n(\beta_2))/D$$
$$D = (\beta_1^2\beta_2 + \beta_2^2\beta_3 + \beta_3^2\beta_1 - \beta_1^2\beta_3 - \beta_2^2\beta_1 - \beta_3^2\beta_2)$$

and initially

$$\beta_1 = 1,\ \beta_2 = \beta_u,\ \beta_3 = \beta_l$$

(3) Step (2) is repeated using the three most recently calculated points until one of the following conditions is met:

(a) Convergence is reached. That is,

$$R_n(\beta) < t R_n(0) \tag{3.10.2}$$

in which $R_n(0)$ is the unbalance at the beginning of the iteration and t is a constant specified by the analyst.

(b) The predicted value of β is beyond the specified limits.

(c) The allowable number of trials is exceeded.

If condition (a) applies, the next iteration is begun from the state corresponding to the latest value of β. If condition (b) or (c) applies, the next iteration is begun from the state with the smallest unbalance.

3.11 UPDATING THE STIFFNESS

3.11.1 General

It has been mentioned that the cost of reforming the stiffness each iteration can be high and that stiffness reformulation is not always necessary for convergence, especially if the change in stiffness between iterations is small. In general, the optimum frequency with which the stiffness matrix is reformed depends on the problem being solved. For this reason, the frequency with which the stiffness matrix is updated is left as a variable to be specified by the analyst. Because the stiffness is not necessarily updated each iteration, the stiffness matrix used

in solving for the displacement increment is not always the current tangent stiffness (although for optimal convergence, it should generally be a close approximation to the current tangent stiffness).

3.11.2 Update Frequency

3.11.2.1 General

The frequency with which the stiffness is updated may be specified at both the step level and the iteration level. Linearization is done only at the specified intervals and is always based on the current state. With appropriate choice of stiffness update frequencies, techniques such as initial stiffness iteration, constant stiffness iteration, and NR iteration can be specified, as follows.

3.11.2.2 Initial Stiffness Iteration

Initial stiffness iteration is obtained if the step frequency is specified greater than the maximum number of steps and if the iteration frequency is specified greater than the maximum number of iterations. Thus, the stiffness is reformed only once, at the beginning of the analysis.

3.11.2.3 Constant Stiffness Iteration

Constant stiffness iteration is obtained if the step frequency is one and the iteration frequency is greater than the maximum number of iterations. The stiffness is then formed only at the beginning of each step.

3.11.2.4 Newton-Raphson Iteration

Newton-Raphson iteration is obtained if both the step frequency and the iteration frequency are equal to one. The stiffness is then reformed every iteration.

3.12 BFGS STIFFNESS MODIFICATION

As a compromise between reforming the stiffness and keeping it constant for a given iteration, an option is included to modify the stiffness matrix. The modification used is the rank two update to the inverse of the stiffness matrix that was introduced in discussion of the BFGS method.

3.13 EVENT PREDICTION

3.13.1 General

In analyses where event occurrences can cause large changes in stiffness, it may be advantageous to predict the next event, advance the solution just beyond it, and then update the state and stiffness. This procedure will keep the solution close to the equilibrium path.

3.13.2 Algorithm

The procedure used to predict events in the advancing phase is as follows.

(1) The displacement increment is calculated.

(2) If it is determined that the calculated displacement will cause an event, the displacement increment is scaled to bring the solution just beyond the first predicted event.

(3) A state determination is performed and $\underline{R}_U$ is calculated.

(4) If it was determined in Step (2) that an event would occur, the state is updated and linearized. Since only a fraction of the step has been applied, the solution returns to Step (1) to apply the remainder of the step. If no event was predicted, the solution enters the correcting phase.

The same procedure is followed for events predicted in the correcting phase, but in Step (1) the increment is chosen to keep the iteration parameter constant.

3.14 STATE DETERMINATION

3.14.1 General

The structure resisting force is calculated by assembling the element resisting forces in the current displaced state. The element forces are obtained by state determination calculations, using the displacement increment accumulated from the reference state to the current state.

3.14.2 Frequency of Updating

The frequency with which the state is updated is specified by the analyst (in terms of number of iterations) and is independent of the stiffness update frequency. This allows the analyst to specify path dependent or path independent state determination, or any scheme in between.

In displacement stepping for softening structures, the load magnitude decreases during iteration. To avoid false unloading of yielded elements, path independent state determination should be used.

3.14.3 Path Dependent State Determination

Path dependent state determination is chosen by specifying that the state be updated every iteration.

3.14.4 Path Independent State Determination

Path independent state determination is chosen by specifying the state update frequency to be larger than the maximum number of iterations. The state at the beginning of the step is then used as the reference state for all state determinations in the step. Path independent state determination has the advantage that the reference states are all converged states.

3.15 EXTERNAL LOAD

The external load is the total load applied to the structure in the current state. It is calculated using the current values of the load pattern magnitudes, allowing for any configuration

dependence.

3.16 UNBALANCE

3.16.1 General

The unbalanced force vector, $\underline{R}_U$, is calculated as the difference between the external applied forces and the structure resisting forces. A norm of the unbalance vector is typically used as the measure of the unbalance of the system. In the ANSR implementation, the Euclidean norm or the infinity norm of $\underline{R}_U$ can be chosen as the unbalance measure.

3.16.2 Exclusion of Rotational DOF

The rotational degrees of freedom may be excluded in calculation of the unbalance. This option is useful because it eliminates combining quantities with different units (i.e., forces and moments).

3.16.3 Other Measures of Unbalance

Other measures of unbalance are possible but are not included in the ANSR implementation. Clough and Bergan [16] suggest using the displacements due to the unbalanced load. Matheis and Strang [3] use an energy measure defined by:

$$R_n = \Delta \underline{r}_a^T \underline{R}_U \tag{3.16.1}$$

in which $\Delta \underline{r}_a$ is the displacement increment in the advancing phase.

The validity of each of these measures as an indication of convergence depends on the state of the structure. When the system is stiff, large load unbalances correspond to small displacement increments. Conversely, when the structure is flexible, small load unbalances can correspond to large displacement increments. This is a disadvantage of basing convergence only on a norm of $\underline{R}_U$. A disadvantage of energy measures is that if the vectors $\underline{R}_U$ and $\Delta \underline{r}_a$ are nearly orthogonal, the energy norm will be small no matter how large the individual vectors may be.

3.16.4 Convergence Rates

Extensive studies have been performed on the convergence properties of Newton-type methods (see, for example, [4,5]). For standard NR iteration, it has been determined that the convergence rate is quadratic for continuously differentiable functions, provided the predicted displacement increment is not too large. The convergence rates for modified Newton and quasi-Newton methods tend to be linear or super-linear.

3.16.5 Predicting Convergence

3.16.5.1 General

Because the methods being used have the property of linear, super-linear, or quadratic convergence, it should be possible to predict whether or not a given iteration sequence is going to converge within the specified number of iterations. It may require a few iterations before a good prediction can be made, and these predictions will not always be correct. However, the advantage of predicting nonconvergence is that it can save computation if it appears that the solution will not converge.

3.16.5.2 Algorithm

The algorithm for convergence prediction is as follows.

Quantities Specified:

Iteration at which prediction begins = I_p.

Weighting factor for convergence rate = w.

Unbalance tolerance = U.

Maximum allowable iterations = n.

(1) For iteration i, a convergence ratio, c_i, is computed by dividing the unbalance at the end of the iteration, R_n^i, by the unbalance at the beginning of the iteration, R_n^{i-1}. That is,

$$c_i = R_n^i / R_n^{i-1} \tag{3.16.2}$$

(2) An estimated convergence rate, C_i, is calculated by taking a weighted average of the individual convergence ratios. The ratio is weighted so that the most recent iterations are counted more heavily. That is,

$$C_i = \frac{c_2 + wc_3 + \cdots + w^{i-1}c_i}{1 + w^2 + \cdots + w^{i-1}} \tag{3.16.3}$$

in which

C_i = i^{th} value of c; and

w^i = w raised to the i^{th} power.

(3) The unbalance is extrapolated using the current unbalance and the estimated convergence rate. The number of iterations, k, required to reach convergence is estimated as:

$$k = \ln(U/R_n^i)/C_i \tag{3.16.4}$$

This value of k is truncated to an integer value, and the number of iterations to convergence is then predicted to be k+1.

(4) If the number of iterations is less than the maximum allowable, the iteration continues. If not, nonconvergence is predicted, and the restepping option is exercised.

Convergence prediction begins on an iteration number specified by the analyst. The use of an iteration other than the second allows the solution to settle down before the prediction begins. In general, the more information available for prediction, the more accurate the prediction will be.

A number of *grace iterations* is allowed so that the solution will continue if the predicted number of iterations to convergence is small, even if the total number is greater than the maximum allowable. The number of grace iterations is specified as a proportion of the number of iterations completed. For example, a proportion of 0.5 means that if 8 iterations have been completed and a prediction of 3 iterations to convergence is made, the solution will continue even if the maximum number of iterations is 10.

4. EXAMPLES

4.1 GENERAL

The solution strategy described in the previous chapter has been tested on a series of four examples. The types of behavior of particular concern in selecting the examples have been (a) buckling, with loss of strength in the post-buckling range, and (b) gap closure, with sudden stiffness increase.

The first three examples have been chosen because they have complex buckling characteristics, and hence, are particularly challenging for the solution strategy. In these examples, the most important aspect of the strategy is the selection of appropriate stepping and direction vectors. The fourth example has several gap elements, which close and produce large changes in stiffness. This type of behavior also challenges the strategy. In this example, the important aspect is the use of the event-to-event option.

4.2 TRUSS STRUCTURE WITH ELASTIC FOLLOW-UP

If a structure buckles locally and subsequently loses strength in the buckled region, it can exhibit the type of behavior shown in Fig. 4.1. If the *unbuckled* length is short, the strength loss can be progressive, as in curve A. However, if the unbuckled length is long, the load-deflection curve can turn back on itself, as in curve B. This reversal occurs when, for any given shortening, δ, in the *buckled* region, the strength loss corresponds to an extension in the *unbuckled* region which exceeds δ.

The response of a structure that exhibits this type of behavior is shown in Fig. 4.2. As the structure buckles, the axial force decreases and the bars outside the buckling region extend. At some distance away from the buckling region, the accumulated effect of the bar extensions is greater than the decrease in length of the buckle. Beyond that point, the displacements reverse direction.

The natural approach to analysis of such a structure would be to control displacement Δ_1 at the loaded point. With this choice, however, no solution can be obtained past point 7 on the

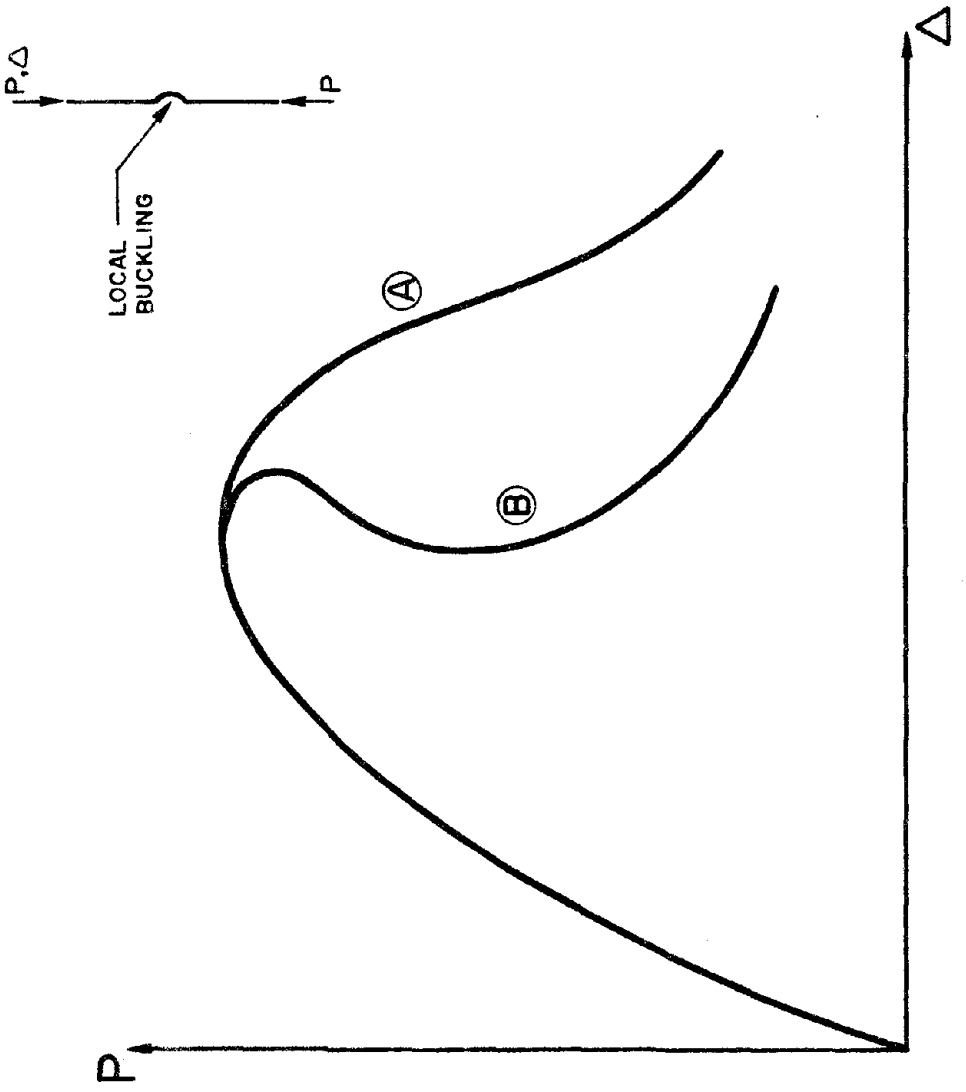

Fig. 4.1. Post-buckling behavior with elastic follow-up.

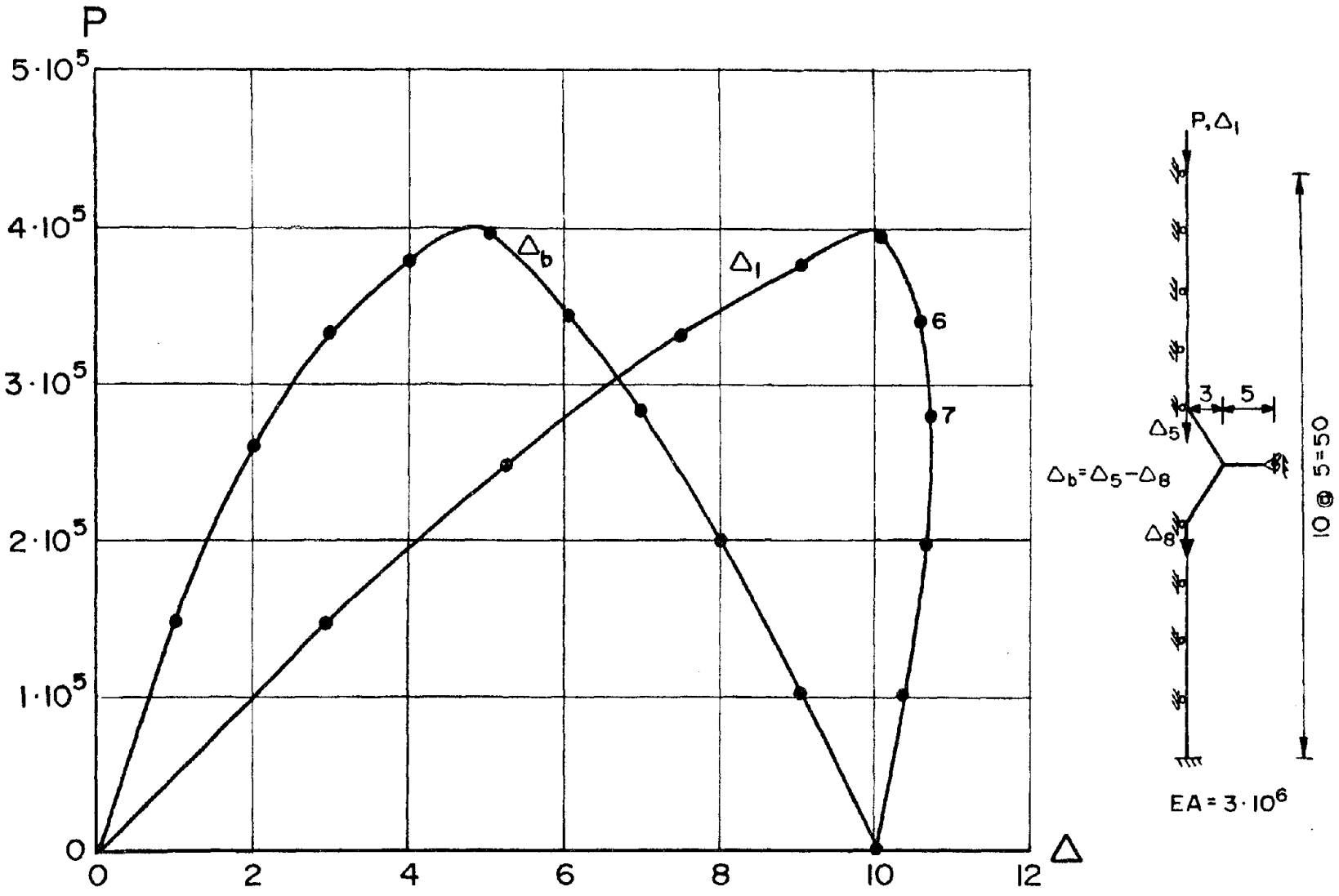

Fig. 4.2. Truss structure with local buckling.

load-deflection curve. If, however, the length of the buckle, Δ_b, is controlled, the solution proceeds without difficulty.

The controlling parameters for the solution were as follows:

(1) The stepping parameter, Δ_b, was increased by 1.0 in each step.

(2) Iteration was performed with Δ_b constant.

(3) The convergence tolerance (Euclidean norm of $\underline{R}_u$) was 0.01.

The input data for ANSR-III is listed in Table 4.1. The maximum number of iterations in any load step was 4.

4.3 SHALLOW ARCH WITH COMPLEX SNAP-THROUGH

Other structures can exhibit load-deflection behavior similar to that of the previous example, but for different reasons. Consider the unsymmetric, shallow, elastic arch shown in Fig. 4.3. The figure shows the load-displacement response at two points, one at the crown and one on the unloaded right-hand side of the arch. The displacement at the crown of the arch, Δ_{18}, undergoes two reversals as the arch snaps through, and hence, is a poor choice for the controlled displacement. The displacement Δ_{26}, however, increases monotonically and is a good choice.

The controlling parameters for the analysis were as follows:

(1) Δ_{26} was increased by 1.0 in each step.

(2) Iteration was performed with Δ_{26} constant.

(3) The convergence tolerance (maximum value of $\underline{R}_u$) was 0.01.

The input data for ANSR-III is listed in Table 4.2. The maximum number of iterations in any load step was 6.

Although the analysis proceeded without difficulty when Δ_{26} was controlled, attempts to obtain a solution by controlling Δ_{18} (or any displacement on the left side of the arch) were inconsistent, and mostly unsuccessful, past point 6 on the load-deflection curve. For example,

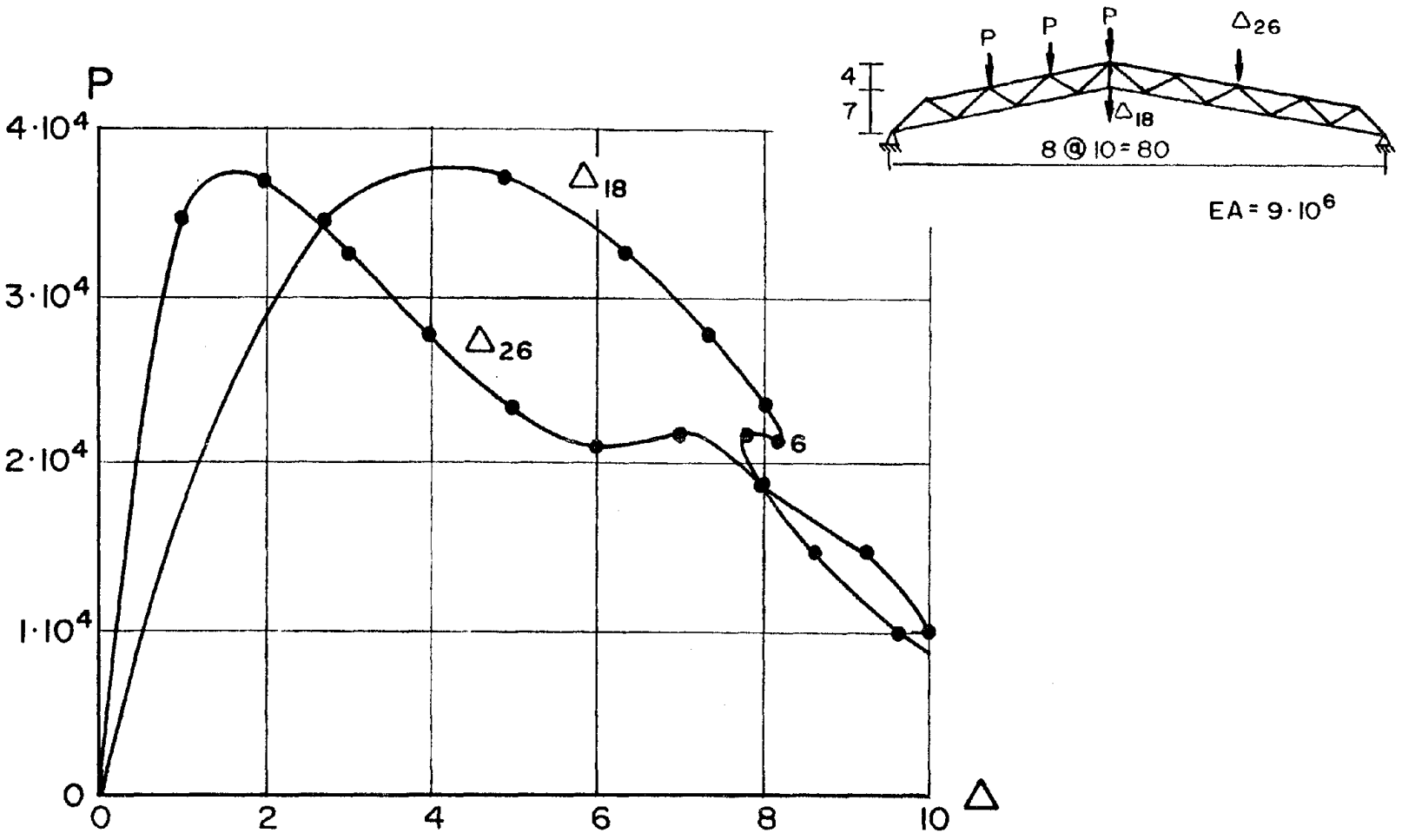

Fig. 4.3. Shallow arch truss with snap-through.

use of Δ_{18} as the controlling parameter converged for a step size of 1.0 (maximum number of iterations = 7) but not for a step size of 0.5.

This example and the previous one show that complex behavior can be analyzed provided that the displacement used to control the solution is well behaved. It may not always be obvious in advance what displacement to control. In most practical cases, however, it should not be too difficult to identify an appropriate displacement.

4.4 TRUSSED TOWER BUCKLING UNDER LATERAL LOADS

In some structures overall structural deformations may dominate initially, but local buckling deformations may then develop and dominate the later response. Because the local buckling deformation is initially negligible, it cannot be used to control the initial response, and some overall displacement must be used. After buckling occurs, however, it may be necessary to control the buckling deformation in order to obtain a solution. That is, different displacements must be controlled at different times.

Fig. 4.4 shows the response of a trussed tower subjected to lateral load. The structure behaves approximately linearly up to a load P = 180, at which time the lowest diagonal brace buckles. For this example the stepping parameter was chosen using the "critical measure" option. The controlling parameters were as follows:

(1) The stepping parameter was chosen as the most critical of the following three displacements: (a) $\Delta_9/6$, (b) $\Delta_4/2$, (c) Δ_3.

(2) The direction parameter was $0.5\ \Delta_4 - \Delta_3$ (that is, essentially the buckling deformation in the lowest brace).

(3) The stepping parameter was held constant during iteration.

(4) The convergence tolerance (Euclidean norm of $\underline{R}_u$) was 0.05.

The input data for ANSR-III is listed in Table 4.3. The maximum number of iterations in any step was 6.

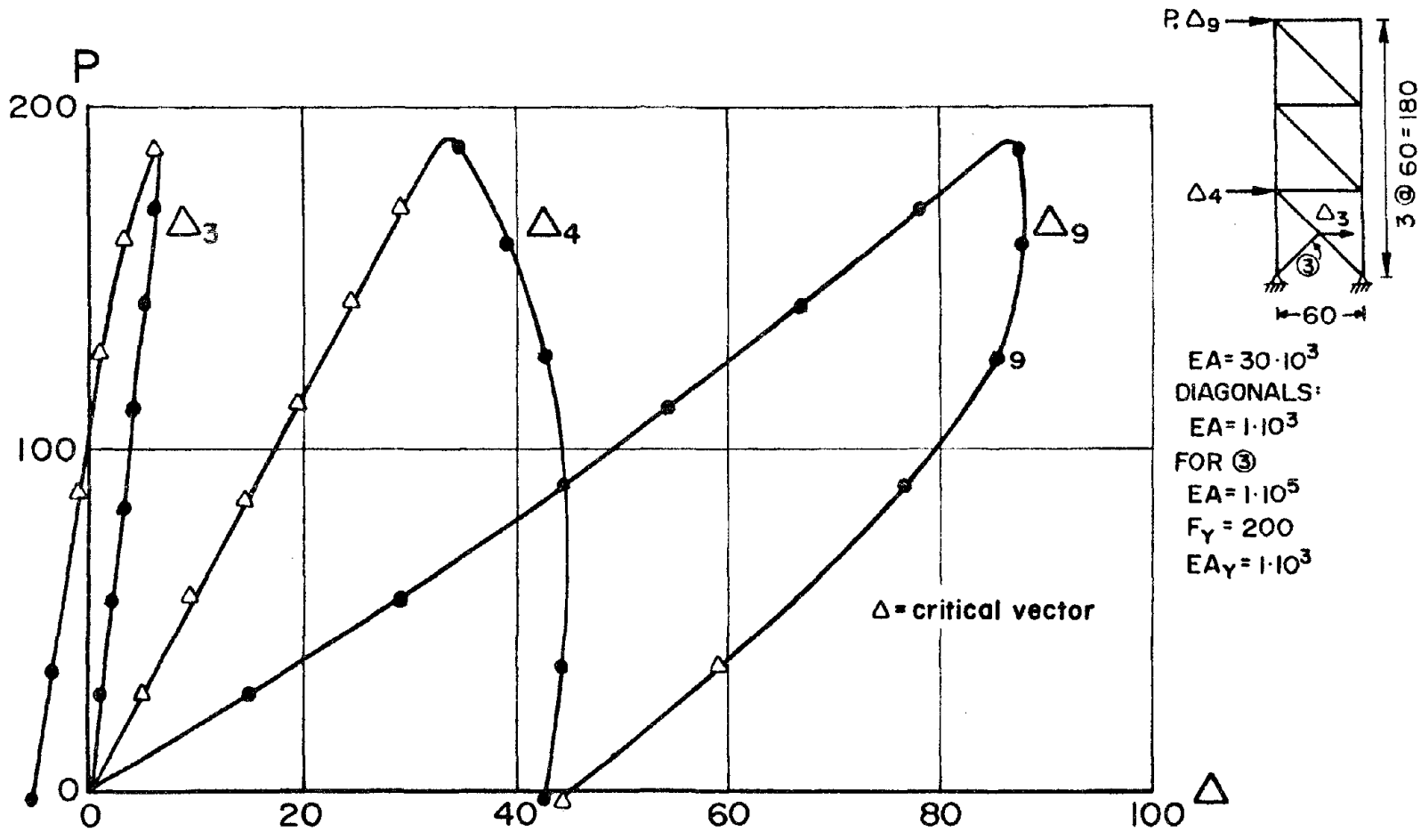

Fig. 4.4. Trussed tower buckling under lateral load.

As the behavior changes, control of the analysis switches between the stepping parameters, as shown in Fig. 4.4. Initially displacement Δ_4 controls, then Δ_3, and finally Δ_9.

This example also indicates the importance of the choice of direction parameter. Because the direction parameter must monotonically increase during the analysis, none of the displacements used for the stepping parameter is suitable. Also, selection of the step direction using Bergan's current stiffness parameter (S_p) is difficult because S_p changes sign by passing through infinity as Δ_9 reverses. At point 9 in the load-deflection relationship, it is not obvious from the previous values of S_p that the next load increment should be negative (i.e., that S_p passed through infinity and not zero).

4.5 PIPE RING CRUSH

Because nonlinear analyses are typically carried out as a series of linear analyses, drastic stiffness changes can make a solution difficult to obtain. One type of behavior that is particularly troublesome is gap closure, which is found, for example, in contact problems.

Fig. 4.5 shows the response of a ring (cut from a section of pipe) subjected to loading between two plattens. As the pipe crushes, the points of contact between the pipe and plattens move outwards, as shown in Fig. 4.6. This is a practical problem taken from an experimental study by Peech et al [17].

The pipe was modelled using shell elements [18]. One-quarter of the ring was modelled with 9 elements, each subtending 10 degrees of arch. The contact between the pipe and the plattens was modelled using gap elements [19], with zero stiffness in tension and very large stiffness in compression. Because of the presence of gap elements, it was important to use the option for event prediction to keep the unbalance from getting excessively large.

The loading was specified as a prescribed displacement of the platten. The controlling parameters were as follows:

(1) Δ_2 was increased in each step.

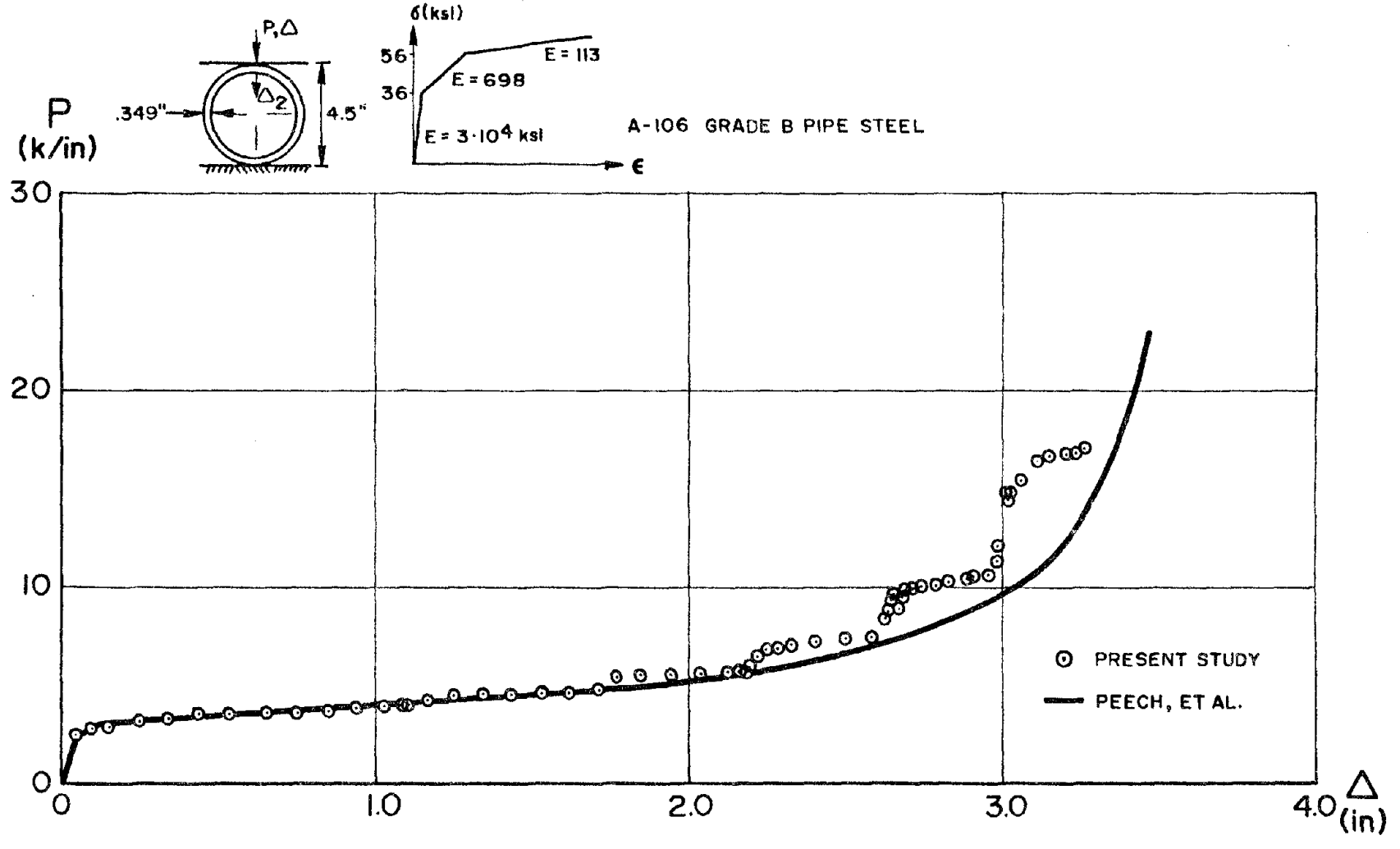

Fig. 4.5. Pipe ring crush.

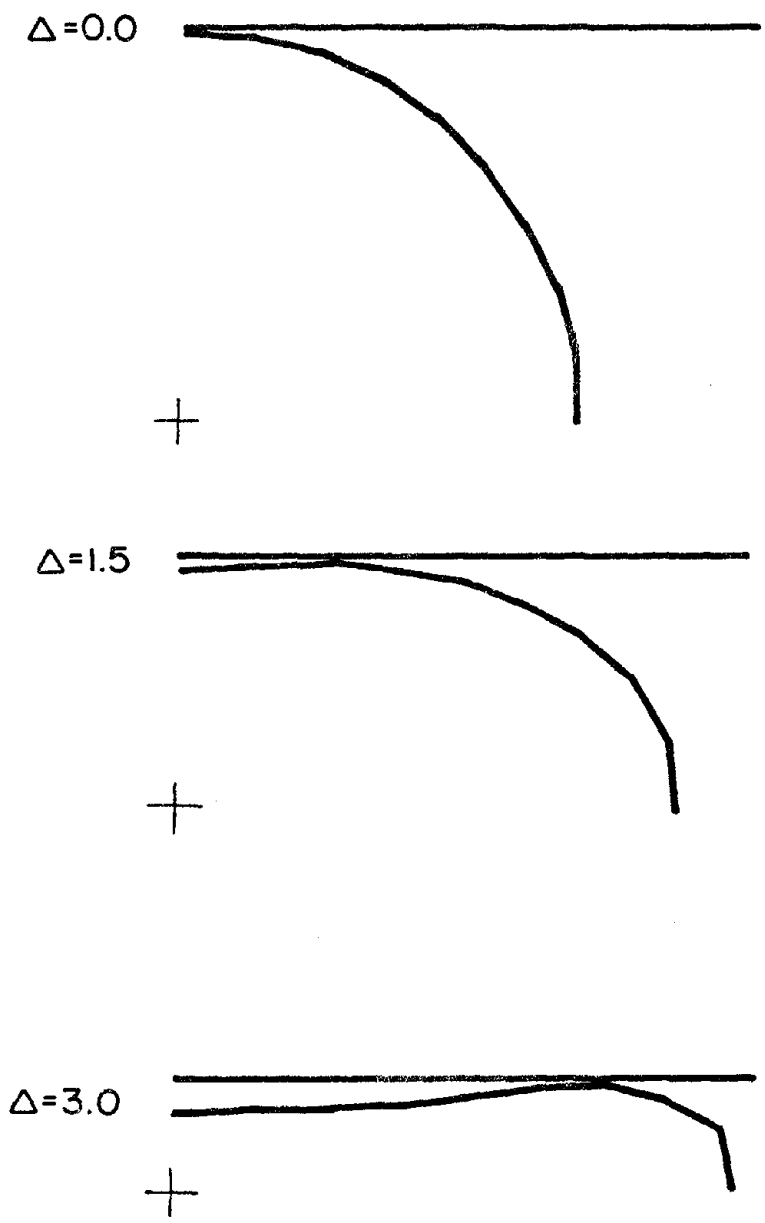

Fig. 4.6. · Calculated deflected shapes for pipe ring crush.

(2) Iteration was performed with Δ_2 constant.

(3) The convergence tolerance (maximum value of $\underline{R}_u$) was 0.02.

(4) If the solution failed to converge in 10 iterations, or if nonconvergence was predicted starting from iteration 5, the step size was multiplied by 1/4 and the step was repeated. If the solution converged in less than 3 iterations, the step size was doubled for the next step.

(5) The event prediction option was used with a limit of 3 events per step. This allowed one gap to close and unload, and another gap to open before event calculation was suppressed.

The input data for ANSR-III are listed in Table 4.4. Although this problem is substantially larger than any of the previous examples, the strategy remained stable over the wide range of stiffnesses encountered in the analysis. The results are plotted in Fig. 4.5, together with experimental data from tests by Peech et al. The agreement is close up to a platten displacement of 1.5 inches. After that discrepancies appear, most probably because of discretization problems (the 10-degree arc elements are not able to capture the behavior accurately when the deformations are very large).

```
star  buckle
nofl
core  symm
  12    12     0     0     0     3     1     0     1
   1          0.0         50.
   2          0.0         45.
   3          0.0         40.
   4          0.0         35.
   5          0.0         30.
   6          3.0         25.
   7          5.0         25.
   8          0.0         20.
   9          0.0         15.
  10          0.0         10.
  11          0.0          5.
  12          0.0          0.
   1      101111    11
   6      001111
  12      111111
01000C      6     7
   1     11     1
   2
   1         3.e6          0.1         1.e9          1.e9
   2         3.e6          .01        2.3e9         2.3e9
   1     1     2     1          1.0          0.0       11100 1.e3        1.e3
   2     2     3     1          1.0          0.0       11100 1.e3        1.e3
   3     3     4     1          1.0          0.0       11100 1.e3        1.e3
   4     4     5     1          1.0          0.0       11100 1.e3        1.e3
   5     5     6     1          1.0          0.0       11100 1.e3        1.e3
   6     6     7     2          1.0          0.0       11100 1.e3        1.e3
   7     6     8     1          1.0          0.0       11100 1.e3        1.e3
   8     8     9     1          1.0          0.0       11100 1.e3        1.e3
   9     9    10     1          1.0          0.0       11100 1.e3        1.e3
```

Table 4.1. ANSR-III input data for first example.

```
  10    10    11     1          1.0          0.0       11100 1.e3         1.e3
  11    11    12     1          1.0          0.0       11100 1.e3         1.e3
   1
   1     vertical end load
   1        0.0      -1.e6

   1    -1
   6
stat
   1      truss with local buckling
   2     3          0.0     1     2          .10    20     0     2     3    10     0
   2       10.0
   5     2         -1.0
   8     2          1.0
   1     1     0    10     2     0
   1     0          .01          .01          0.0     1          3.0          0.0          0.5
        .25         .01     0     3          2.0          4.0     0
   1.0
none
stop
```

Table 4.1. (cont.)

```
star   truss arch
nofl
core sym
  18     9     4     0     0     3     0     0     1
   1          0.0          0.0
   2          5.0          5.0
   7         30.0          6.0
   8         35.0         11.0
   9         35.0          7.0
  10         40.06.22222222
  11         45.09.44444444
  17         75.04.77777778
  18         80.0          0.0
   1     7     2     2
   2     8     2     2
  11    17     2     2
  10    18     3     2
   1     001111    18
   1     111111
  18     111111
   1    33     1
   1
   1          9.e6          0.1       1.e15       1.e15
   1     1     2     1          1.0          0.0       11100 1.e3        1.e3
   2     1     3     1          1.0          0.0       11000 1.e3        1.e3
   3     2     3     1          1.0          0.0       11000 1.e3        1.e3
   4     2     4     1          1.0          0.0       11000 1.e3        1.e3
   5     3     4     1          1.0          0.0       11000 1.e3        1.e3
   6     3     5     1          1.0          0.0       11000 1.e3        1.e3
   7     4     5     1          1.0          0.0       11000 1.e3        1.e3
   8     4     6     1          1.0          0.0       11000 1.e3        1.e3
   9     5     6     1          1.0          0.0       11000 1.e3        1.e3
```

Table 4.2. ANSR-III input data for second example.

```
 10     5     7     1        1.0        0.0      11000 1.e3       1.e3
 11     6     7     1        1.0        0.0      11000 1.e3       1.e3
 12     6     8     1        1.0        0.0      11000 1.e3       1.e3
 13     7     8     1        1.0        0.0      11000 1.e3       1.e3
 14     7     9     1        1.0        0.0      11000 1.e3       1.e3
 15     8     9     1        1.0        0.0      11000 1.e3       1.e3
 16     9    10     1        1.0        0.0      11000 1.e3       1.e3
 17     8    10     1        1.0        0.0      11000 1.e3       1.e3
 18     8    11     1        1.0        0.0      11000 1.e3       1.e3
 19    10    11     1        1.0        0.0      11000 1.e3       1.e3
 20    10    12     1        1.0        0.0      11000 1.e3       1.e3
 21    11    12     1        1.0        0.0      11000 1.e3       1.e3
 22    11    13     1        1.0        0.0      11000 1.e3       1.e3
 23    12    13     1        1.0        0.0      11000 1.e3       1.e3
 24    12    14     1        1.0        0.0      11000 1.e3       1.e3
 25    13    14     1        1.0        0.0      11000 1.e3       1.e3
 26    13    15     1        1.0        0.0      11000 1.e3       1.e3
 27    14    15     1        1.0        0.0      11000 1.e3       1.e3
 28    14    16     1        1.0        0.0      11000 1.e3       1.e3
 29    15    16     1        1.0        0.0      11000 1.e3       1.e3
 30    15    17     1        1.0        0.0      11000 1.e3       1.e3
 31    16    17     1        1.0        0.0      11000 1.e3       1.e3
 32    16    18     1        1.0        0.0      11000 1.e3       1.e3
 33    17    18     1        1.0        0.0      11000 1.e3       1.e3
  1
  3     gravity load
  4          0.0     -1.0e5
  6          0.0     -1.0e5
  8          0.0     -1.0e5

-1    -1    0
```

Table 4.2. (cont.)

```
stat
   1      truss arch
   2     3                1     2         .10    10     0     2     3     7
   1       10.
  13     2        -1.0
   1     1     0    10     2     0
   0     0          .01          .01        0.0     1          3.0          0.0          0.5
        .25          0.0     0     3         2.0          4.0     0
  10.0
none
stop
```

Table 4.2. (cont.)

```
star   truss tower
nofl
core  symm
   9     9     0     0     0     2     0     0     1
   1          0.0           0.0
   2         60.0           0.0
   3         30.0          30.0
   4          0.0          60.0
   5         60.0          60.0
   6          0.0         120.0
   7         60.0         120.0
   8          0.0         180.0
   9         60.0         180.0
   1     111111     2
   3     001111     9
   1    14     1
   4
   1        3.e4          0.1       1.e15       1.e15
   2        1.e3          0.1       1.e15       1.e15
   3        1.e5        1.e-2       2.0e2       2.0e2
   4       1.0e3           .1       1.5e5       1.5e5
   1     1     4     1         1.0         0.0      11100 1.e3        1.e3
   2     4     3     2         1.0         0.0      11100 1.e3        1.e3
   3     1     3     3         1.0         0.0      11100 1.e3        1.e3
   4     3     2     2         1.0         0.0      11100 1.e3        1.e3
   5     2     5     1         1.0         0.0      11100 1.e3        1.e3
   6     4     5     1         1.0         0.0      11100 1.e3        1.e3
   7     4     6     1         1.0         0.0      11100 1.e3        1.e3
   8     6     5     4         1.0         0.0      11100 1.e3        1.e3
   9     5     7     1         1.0         0.0      11100 1.e3        1.e3
  10     6     7     1         1.0         0.0      11100 1.e3        1.e3
  11     6     8     1         1.0         0.0      11100 1.e3        1.e3
```

Table 4.3. ANSR-III input data for third example.

```
  12     6     7     4          1.0          0.0      11100 1.e3         1.e3
  13     7     9     1          1.0          0.0      11100 1.e3         1.e3
  14     6     9     1          1.0          0.0      11100 1.e3         1.e3
   1
   1    lateral load
   8     1.0e3

  -1    -1     0
stat
   1       truss tower
   2     3                  3     3          .05   20     0     2     3     7
   1      300.0
   8     1          1.0
   1      100.0
   4     1          1.0
   1      50.0
   3     1          1.0
   2
   4     1          .5
   3     1         -1.
   1     1     0    10     2     0
   1     0          .05          .05          0.0    1          3.0          0.0          0.5
       .25          0.0     1     3          2.0          4.0     5
  1.0
none
stop
```

Table 4.3. (cont.)

```
star        pipe crush
newf     1     1     6step
ccre  symm
  47    47     0     0     0    23    28     0     3     0
   1          .0000          .0000          .0000
   2          .0000             0.0         2.0755
   3          .0000             0.0          .1745
   4          .0000             1.0         2.0755
   5          .0000             0.0          .1745
   6          .3604             0.0          .0000
   7          .3604             0.0         2.0440
   8          .0303             0.0          .1718
   9          .3604             1.0         2.0440
  10          .0303             0.0          .1718
  11          .7099             0.0          .0
  12          .7099             0.0         1.9503
  13          .0597             0.0          .1640
  14          .7099             1.0         1.9503
  15          .0597             0.0          .1640
  16         1.0378             0.0          .0
  17         1.0378             0.0         1.7974
  18          .0873             0.0          .1511
  19         1.0378             1.0         1.7974
  20          .0873             0.0          .1511
  21         1.3340             0.0          .0
  22         1.3340             0.0         1.5899
  23          .1122             0.0          .1337
  24         1.3340             1.0         1.5899
  25          .1122             0.0          .1337
  26         1.5899             0.0          0.0
  27         1.5899             0.0         1.334
  28          .1337             0.0          .1122
  29         1.5899             1.0         1.334
  30          .1337             0.0          .1122
  31         1.7974             0.0          0.0
  32         1.7974             0.0         1.0378
  33          .1511             0.0          .0873
  34         1.7974             1.0         1.0378
  35          .1511             0.0          .0873
  36         1.9503             0.0          .7099
  37          .1640             0.0          .0597
  38         1.9503             1.0          .7099
  39          .1640             0.0          .0597
  40         2.0440             0.0          .3604
  41          .1718             0.0          .0303
  42         2.0440             1.0          .3604
  43          .1718             0.0          .0303
  44         2.0755             0.0            0.0
  45          .1745             0.0            0.0
  46         2.0755             1.0            0.0
  47          .1745             0.0            0.0
```

Table 4.4. ANSR-III input data for fourth example.

1	110111	3									
4	100111										
5	110111										
6	010111	8									
9	000111										
10	010111	13									
14	000111										
15	010111	18									
19	000111										
20	010111	23									
24	000111										
25	010111	28									
29	000111										
30	010111	33									
34	000111										
35	010111	37									
38	000111										
39	010111	41									
42	000111										
43	010111										
44	011111	45									
46	001111										
47	011111										
010000	4	9	14	19	24	29	34	38	42	46	
001000	1	6	11	16	21	26	31				
001000	2	4									
001000	3	5									
100000	6	7	9								
001000	7	9									
101000	8	10									
100000	11	12	14								
001000	12	14									
101000	13	15									
100000	16	17	19								
001000	17	19									
101000	18	20									
100000	21	22	24								
001000	22	24									
101000	23	25									
100000	26	27	29								
001000	27	29									
101000	28	30									
100000	31	32	34								
001000	32	34									
101000	33	35									
101000	36	38									
101000	37	39									
101000	40	42									
101000	41	43									
100000	44	46									
100000	45	47									

Table 4.4. (cont.)

```
1     1     1
1
1     1.0e-5          .10       1.e15       1.e15
1     1     2     1          1.0          0.0       00100       1.0e3    1000.
5     7     1
7     1
1     2      -1.e-5      .00001
6.0e+3      6.0e+3
.10e15      .10e15
2     2       .0315      .00001
 6.0e3       6.0e3
.10e15      .10e15
3     2      .12516      .00001
 6.0e3       6.0e3
.10e15      .10e15
4     2       .2781      .00001
 6.0e3       6.0e3
.10e15      .10e15
5     2       .4856      .00001
 6.0e3       6.0e3
.10e15      .10e15
6     2       .7412      .00001
 6.0e3       6.0e3
.10e15      .10e15
7     2      1.0378      .00001
 6.0e3       6.0e3
.10e15      .10e15
1     1     2     1     0     1     0          1.0          1.0
2     6     7     2     0     1     0          1.0          1.0
3    11    12     3     0     1     0          1.0          1.0
4    16    17     4     0     1     0          1.0          1.0
5    21    22     5     0     1     0          1.0          1.0
6    26    27     6     0     1     0          1.0          1.0
7    31    32     7     0     1     0          1.0          1.0
```

Table 4.4. (cont.)

```
2     9     1
1     8   225   111     1     2     0     1
1       .27e5         .30
697.69      113.15
.360e2      .560e2
  .010          .01
1     2     7     9     4     3     8    10     5     1
2     7    12    14     9     8    13    15    10     1
3    12    17    19    14    13    18    20    15     1
4    17    22    24    19    18    23    25    20     1
5    22    27    29    24    23    28    30    25     1
6    27    32    34    29    28    33    35    30     1
7    32    36    38    34    33    37    39    35     1
8    36    40    42    38    37    41    43    39     1
9    40    44    46    42    41    45    47    43     1
0     0     0     0     1
1           start the crush
1     3       -.1e1

18    1    19
 7    8    12    13    17    18    22    23    27    28    32    33    36    37    40    41
44   45
 4
 1    2     3     7     8    12    13    17    18    22    23    27    28    32    33    36
37   40    41
```

Table 4.4. (cont.)

```
stat
   1                    pipe crush
   2     3        1.0     1     2      .0250    70     3     2     3    10     0
   1        1.0     1
   2     3        -1.0
   1     1     0   10     2     1
   0     0        .20        .20        0.0     1        3.0        0.0        0.5
        .25       .01     0     3        2.0        8.0     5
   4.0
none
stop
```

Table 4.4. (cont.)

5. CONCLUSIONS

5.1 OBJECTIVES ACHIEVED

The objectives of the research, as outlined in Section 1.2, have been substantially fulfilled, as follows.

(1) The tasks and tools found in static nonlinear analysis have been described using a consistent terminology.

(2) By generalizing Newton-Raphson iteration, a framework was developed which allows comparison of different existing strategies and assists in the development of new strategies.

(3) A general strategy has been developed and incorporated into ANSR-III. This strategy can be applied to the solution of a wide variety of problems, yet is reasonably easy to use.

(4) The strategy has been shown to be stable and efficient for several problems that would be troublesome for most existing strategies.

5.2 WEAKNESSES

Although substantial progress has been made, weaknesses still exist in the proposed strategy. Some of these are as follows.

(1) The strategy still requires skill on the part of the analyst, particularly in choosing the stepping and direction parameters. Although it should not be difficult to choose appropriate parameters for many practical analyses, automated schemes for choosing the parameters would be a convenience.

(2) Application of the solution strategy has been limited to small problems. Resources were not available to test the strategy on large-scale practical problems.

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APPENDIX A

J. STATIC ANALYSIS

Omit entire Section J if IANTYP ≠ STAT.

J1. TITLE

COLUMNS	NOTE	NAME	DATA
1 - 5(I)	J.1	NSAS	No. of analysis segments
9 - 80(A)			Optional analysis title.

J2. SEGMENT SPECIFICATION

NSAS sets of cards, one set for each analysis segment.

J2(a). SOLUTION CONTROL

COLUMNS	NOTE	NAME	DATA
5(I)		KTYS	Step type: (a) 1: Load stepping. (b) 2: Displacement stepping.
10(I)		KPAS	Stepping parameter (KTYS = 2 only): (a) 1: Euclidean norm. (b) 2: Maximum value. (c) 3: Specified stepping vector(s)
11 - 20(F)		DISI	Segment value (KPAS = 1,2 only).
25(I)		NVES	No. of stepping vectors (KPAS = 3 only). Max = 4. Default = 1.
30(I)		KDIR	Step direction code: (a) 1: Increase load magnitude each step. (b) 2: Increase scalar displacement using (critical) stepping vector (KPAS = 3 only). (c) 3: Increase scalar displacement using direction vector. (d) 4: Determine sign of load increment using Bergan's current stiffness parameter.
31 - 40(F)		STEPI	Initial step factor for segment. Must be between 0.0 and 1.0.
41 - 45(I)		MAXST	Maximum no. of steps. Analysis stops if this no. is exceeded. Default = no limit.
46 - 50(I)		IEVENT	Element event code: (a) 0: No event check. (b) 1: Event check, no limit on number of events. (c) N(>1): Check events, up to N events in any step.
55(I)		KTYI	Iteration type: (a) 1: Constant load. (b) 2: Constant scalar displacement.

60(I)	KPAI	Iteration parameter (KTYI = 2 only). (a) 1: Constant work. (b) 2: Constant "arc length". (c) 3: Critical stepping vector (KPAS must = 3). (d) 4: Iteration vector.
65(I)	MAXIT	Max. no. of iterations per step.
70(I)	IQUIT	Termination code: (a) 0: If MAXIT is exceeded, restep. Include card J2(e). (b) 1: If MAXIT is exceeded, stop. (c) 2: If MAXIT is exceeded, continue.

J2(b). SPECIFIED VECTORS

NVES sets of cards defining the stepping vectors if KPAS = 3. Plus one set of cards defining the direction vector if KDIR = 3. Plus one set of cards defining the iteration vector if KPAI = 4.

Each set consists of a vector title card plus one or more cards defining the terms of the vector. The number of terms for any vector must not exceed 20.

J2(b) (i) VECTOR TITLE

COLUMNS	NOTE	NAME	DATA
1 - 5(I)		NDGC	No. of terms in this vector (max. = 20).
6 - 15(F)		FACI	Segment value (stepping vectors only).
16 - 80(A)			Optional title.

J2(b) (ii) VECTOR DEFINITION

As many cards as needed to specify NDGC terms, 4 terms per card.

COLUMNS	NOTE	NAME	DATA
1 - 5(I)			Node no.
10(I)			D.O.F. number (1-6).
11 - 20(F)			Weighting factor. Default = 1.0.
21 - 40			Similar data for second term (2I5,F10.0).
41 - 60			Similar data for third term (2I5,F10.0).
61 - 80			Similar data for fourth term (2I5,F10.0).

J2(c). STIFFNESS, STATE AND OUTPUT CONTROL

COLUMNS	NOTE	NAME	DATA
1 - 5(I)		KUFS	Step control for stiffness update. Stiffness is updated every KUFS steps. Default = 1.
6 - 10(I)		KUFI	Iteration control for stiffness update. Stiffness is updated every KUFI iterations. Default = 1.
15(I)		KBFGS	BFGS code: (a) 0: No BFGS. (b) 1: BFGS between stiffness updates.
16 - 20(I)		KUFSD	Iteration control for state update. State is updated every KUFSD iterations. State is automatically updated at the end of every step.
25(I)		KPRIN	Results print code: (a) 0: No printed output. (b) 1: Print at end of analysis segment only. (c) 2: Print every step. (d) 3: Print every iteration.
30(I)		KSAVE	Results save code: (a) 0: Do not save on restart file. (b) 1: Save on restart file (at the end of the analysis segment).

J2(d). CONVERGENCE CRITERIA

COLUMNS	NOTE	NAME	DATA
5(I)		INRM	Unbalance type: (a) 0: Max. value. (b) 1: Euclidean norm.
10(I)		MOCD	Moment code for unbalance: (a) 0: Ignore moments. (b) 1: Include moments.
11 - 20(F)		TOL	Unbalance tolerance, for all except last step of segment.
21 - 30(F)		TOLF	Unbalance tolerance for last step. Default = TOL.
31 - 40(F)		UNBLS	Unbalance for step size scaling (must be greater than TOL). (a) 0.0: No scaling. (b) >0.0: Linear scaling of step size until unbalance is less than UNBLS.
41 - 45(I)		MAXLS	Maximum no. of line searches for any iteration. Default = none. (Leave rest of card blank.)
46 - 55(F)		UPLS	Upper line search limit (multiple of calculated displacement). Must be > 1.0.
56 - 65(F)		BMLS	Lower line search limit (multiple of calculated displacement). Must be between 0.0 and 1.0.
66 - 75(F)		TOLS	Convergence tolerance for line search (multiple of initial unbalance for the iteration). Must be between 0.0 and 1.0.

J2(e). RESTEPPING CRITERIA (omit if IQUIT not 0)

COLUMNS	NOTE	NAME	DATA
1 - 10(F)		DNFAC	Multiplier for step factor reduction. If MAXIT is exceeded step factor is multiplied by DNFAC. Default = 0.25.
11 - 20(F)		STPMIN	Minimum allowable step factor, as a multiple of the initial step factor. Default = 1/64 = .0156.
21 - 25(I)		LQUIT	Termination code: (a) 0: If STPMIN is reached, continue with step factor STPMIN. (b) N: If more than N attempts are made to reduce the step factor below STPMIN, stop the analysis.
26 - 30(I)		MINIT	Lower limit on the number of iterations for step factor increase. If convergence is obtained in MINIT or fewer iterations, multiply step factor by UPFAC.
31 - 40(F)		UPFAC	Multiplier for step factor increase. Default = 4.
41 - 50(F)		STPMAX	Maximum allowable step factor as a multiple of the initial step factor. Default = 1.
55(I)		ITPR	Convergence prediction code: (a) 0: No prediction. (b) N: Start prediction at iteration N.
56 - 65(F)		GRACE	Grace ratio. If predicted no. of additional iterations for convergence is less than (GRACE)*(current iteration no.) then iteration continues even if MAXIT is exceeded. Default = 0.5.
66 - 75(F)		WTFAC	Weighting factor for iteration prediction. The convergence ratio of the current iteration is weighted 1.0, at the previous iteration 1.0/WTFAC, etc. Default = 1.0.

J2(f). STATIC LOAD PATTERN APPLICATION

The total number of static load patterns is

NPAT = NPNF + NPFF + NPTP + NPP4 + NPND

from Card F1. Static load patterns are numbered in the sequence that they are input.

Use as many cards as needed to specify NPAT scale factors (one per pattern), 8 per card in 8F10.0 format. To omit any pattern, specify a zero or blank scale factor.

USER GUIDE NOTES

NOTE J.1

Static loads and imposed displacements can be applied in a number of static load **segments.** Each segment is obtained by combining static force and/or imposed displacement **patterns.** Each segment can be applied in a number of **steps** that are either load-controlled or displacement-controlled. For each step the solution is typically found in a number of **iterations,** the iteration path depending on the solution strategy selected. The results at the end of any segment may be saved on the restart file.

A different solution strategy may be specified for each new load segment. The scale factors for the load and displacement patterns may also be different from one segment to the next.

EARTHQUAKE ENGINEERING RESEARCH CENTER REPORTS

NOTE: Numbers in parentheses are Accession Numbers assigned by the National Technical Information Service; these are followed by a price code. Copies of the reports may be ordered from the National Technical Information Service, 5285 Port Royal Road, Springfield, Virginia, 22161. Accession Numbers should be quoted on orders for reports (PB --- ---) and remittance must accompany each order. Reports without this information were not available at time of printing. The complete list of EERC reports (from EERC 67-1) is available upon request from the Earthquake Engineering Research Center, University of California, Berkeley, 47th Street and Hoffman Boulevard, Richmond, California 94804.

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